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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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Yerkes Observatory of the University of Chicago

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Ryerson Physical Laboratory of the University of Chicago

MARCH 1922

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PLATE I

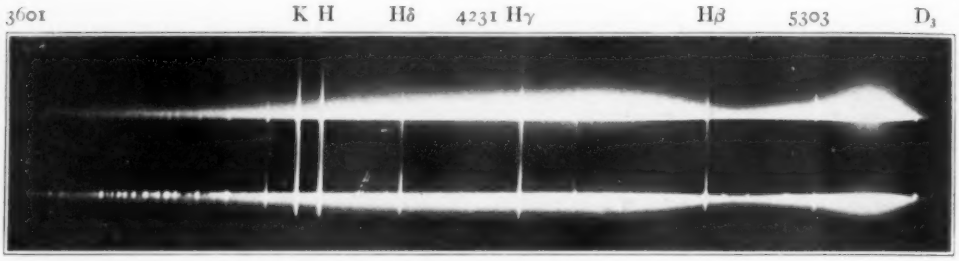

Fig. 1.—General spectrum of the corona. Single prism. Top W

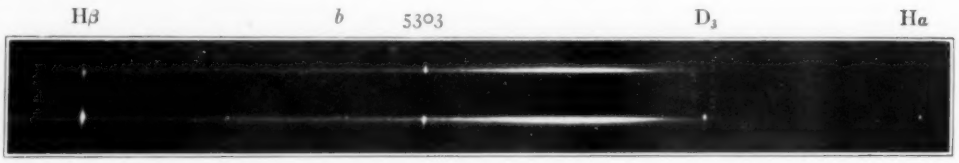

Fig. 2.—Spectrum of inner corona. Three prisms. Top E

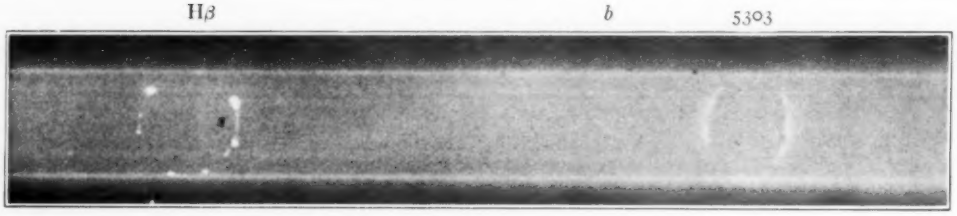

Fig. 3.—Hβ ring and green coronium ring. Top N

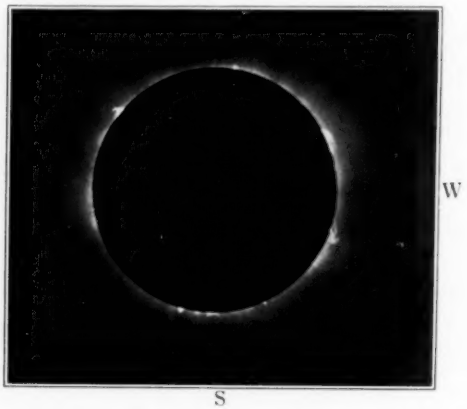

Fig. 4.—Prominences and inner corona with 39-foot camera.

Eclipse of June 8, 1918

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THE SPECTRUM OF THE CORONA AS OBSERVED BY THE EXPEDITION FROM THE LOWELL OBSERVATORY AT THE TOTAL ECLIPSE OF JUNE 8, 1918

By V. M. SLIPHER

ABSTRACT

Spectrum of corona observed at total eclipse of June 8, 1918.—The Lowell Observatory party, located near Syracuse, Kansas, obtained photographs with the following four spectrographs: (1) with single prism, 49 cm camera, slit E–W; (2) with single prism, 8.3 cm camera, slit N–S, then E–W; (3) with three prisms, 9-inch camera, slit E–W; (4) with three prisms, edges N–S, 47 cm camera, no slit. Weather conditions somewhat interfered with preparations; and thin haze veiled the sun during the eclipse, causing some scattering of light that rendered less certain the interpretation of some of the results. Yet useful plates were obtained with all the instruments. These show the usual prominence and chromospheric lines, the chief lines of coronium, the continuous spectrum of the corona, with weak dark lines whose origin is not quite certain; and strips of continuous spectrum coincident with certain prominences and sections of the inner corona uncovered by depressions of the moon's limb.

Distribution of coronium.—The coronium line λ5303.0 indicates that this substance extended about a solar diameter above the sun's surface. The slitless spectra show many condensations in the green coronium ring. These are not related to the prominences. Coronium was fairly abundant generally along the sun's limb under those major extensions of the outer corona, and scarce where the coronal rays typical of the polar regions occupied the limb.

The Lowell Observatory station for observing the solar eclipse of June 8, 1918, was located about eight miles north and one mile east of Syracuse, Kansas. Points along the path to the west of Denver offered somewhat better conditions as regards weather prospects, higher altitude of the sun and somewhat longer period of totality; but the advantages of accessibility and wider distribution of the expeditions along the eclipse path were sufficiently weighty to determine upon a station where the main line of the

Santa Fe Railroad was intersected by the eclipse path in western Kansas. This region is only one day by direct train from Flagstaff. Thus shipments of apparatus could be depended upon to go through more promptly and with less danger of injury than where changes in transit are involved. Then, too, this choice placed the Lowell station the farthest eastward of any of those prepared to undertake general observations of the eclipse.

In the autumn of 1917, Messrs. C. O. Lampland and E. C. Slipher looked over the general conditions in western Kansas with a view to finding a station site, and early in May, 1918, Mr. E. C. Slipher went to Syracuse, chose a site, found its latitude and the meridian and made all necessary arrangements, secured the needed help and materials, and went forward with the preparations. Others proceeded to Syracuse later as their rôles required, until the membership of the expedition was constituted thus: Mr. and Mrs. V. M. Slipher, Mr. and Mrs. C. O. Lampland, Mr. E. C. Slipher, Mr. Stanley Sykes, and Mr. E. C. Mills, of the Lowell Observatory; Mr. George R. Agassiz, of Boston, Massachusetts, and Mr. and Mrs. O. H. Truman, of the University of Iowa; and, on June 8, Mrs. Percival Lowell joined the expedition for the eclipse.

During the last week preceding eclipse day the weather was, for the most part, unfavorable. There were frequent showers and the sky was clouded much of the time, day and night. From what had been learned from weather records it had been expected that some clouds and rain would be encountered, but not so much as was experienced the last week. Thus the preparations and rehearsals could not be completed in the manner originally planned.

On eclipse day the sky at sunrise was only partly clear, and soon became completely overcast. However, in the late forenoon the clouds dissipated and by mid-day the sky was mostly clear. Throughout the afternoon some veil-like clouds of haze were present and at eclipse time one of these covered the region of the sun. This haze somewhat reduced the light and rather seriously limited the effectiveness of the small-scale cameras in recording the extreme outer extensions of the corona, by its direct action, and also by its scattering the light of the prominences and bright inner corona over the faint outer parts of the corona. However, although the

haze must have affected to some extent the photographs of the inner corona and the spectrograms, its ill effects were not always manifest. Indeed, the two large-scale cameras, particularly, secured a number of negatives of the brighter parts of the corona that in comparison with other eclipse results are seen to be of a high degree of excellence. This valuable and extensive material will be published later by Messrs. Lampland and E. C. Slipher, who were responsible for that part of the eclipse program and deserve full credit for the success of the results.

It was thought wise to confine the studies of the spectrum to that of the corona. For this work serviceable instruments could be arranged from the stellar and nebular spectrographic equipment of the observatory. High linear dispersion was not considered vital in the observations to be undertaken and in arranging the different spectrographs questions of choice between dispersion and light-power were decided to favor light-power, in order that the instruments might be quick enough under good conditions to get a strong record of the spectrum and still to get some record even should the conditions be poor. The mountings of the instruments were made largely of wood, but metal was employed where most needed to give the optical parts proper support. The weather at Syracuse made clear the weakness of wood and the wisdom of making all essential parts of metal. At the eclipse camp Mr. O. H. Truman assisted me for several days before the eclipse, and he and Mrs. Slipher shared with me the operation of the instruments during the eclipse.

The single-prism spectrograph.—A spectrograph of one 63° flint glass prism and a 380-mm camera of 33-mm aperture was used in recording the general spectrum of the inner corona from D to the ultra-violet. It was supplied with an image-forming lens of about the same aperture and focus as the collimator, which was of 490-mm focus and 32-mm aperture. The slit was east and west over the sun's diameter and long enough to include the corona to some distance off the east and west limbs of the sun. The exposure ran from count 2 to 78 of the 83 seconds of the total phase. A Cramer Isochromatic plate was used, and recorded the spectrum from D_3 to about λ 3650 in the ultra-violet.

The exposure began soon enough after second contact to register a great number of chromospheric bright lines on the east edge of the sun, but it stopped too early to show these on the west edge.

The prominence lines of calcium, hydrogen, and helium are long, especially so those of calcium. The chief coronium line, that near λ 5303.0 is also strong and, like the lines mentioned above, it extends across the space occupied by the moon's image, owing to scattering of the light in the earth's atmosphere. The coronium line near λ 4231 is present and also weak impressions of the lines λ 4086 and 3601. The continuous spectrum of the inner corona extends from D almost to λ 3650, with the general variations of intensity that are to be expected from the variations in color sensitiveness of the photographic plate used. In its strongest portion—in the blue—it is recorded outward from the sun's east limb to a distance of about 10 minutes of arc and to about 14 minutes from the west limb. In the outer margin of the continuous spectrum there are irregularities, especially between Hδ and λ 4500, which I am more inclined to believe are the weak beginnings of the dark-line solar spectrum rather than that they are bright lines of a true coronal origin. This plate is reproduced in Figure 1, Plate I.

Low-dispersion spectrograph.—A slit spectrograph of high light-power having a 66° prism of dense but clear flint glass and a camera of 35-mm aperture and 83-mm focus, was used in photographing the spectrum of the corona to as great distances from the sun's surface as possible. This spectrograph with its image-forming lens was mounted so that it could be rotated through 90° in order to obtain two spectrograms: one with the slit N–S, for the outer corona off the north and south limbs of the sun; the other, with slit E–W, to cover the corona to the east and west of the sun. Thus, with a long slit it was planned to compare the spectra of two of the most dissimilar parts of the corona, the polar, and the equatorial regions. The slit was long enough to cover the corona out on each side to about 1.2 times the solar diameter. Unfortunately we did not have time, under the weather conditions, to perfect the adjustments for rotation so as to have the slit cover the desired portions of the corona for the polar and the equatorial comparisons.

In consequence, for the first exposure the slit which was east and west crossed the moon's image near the south limb. The exposure started two seconds after the second contact and continued during the first half of totality. Then at mid-totality the instrument was rotated through 90°—which brought the slit north and south and about 10′ west of the sun's west limb—the plate was shifted and the second exposure continued up to within about two seconds of third contact.

The plate used was a Cramer Trichromatic. It has a good impression of the continuous spectrum, over the full length of the slit, and from the violet down to the D line. The exposure with the slit crossing the sun's image near the south limb shows many prominence and chromospheric emission lines, including the red line of hydrogen. The green coronium line is clearly recorded by the first exposure and faintly by the second. This record of the coronium line will be of interest later in connection with the slitless spectrogram. A few of the strongest groups of solar absorption lines are discernible toward the edges of the spectrum of the first exposure and traceable across the full width of the second one.

The variation of intensity of these spectra across their width is in full accord with what the direct photographs indicate for light intensities along lines over the corona occupied by the slit: i.e., the first exposure slit E–W, shows more extension and density to the west than to the east of the sun, and the second more uniform density across its width. The ends of the slit for the first exposure reached distances of about 1.2 diameters from the sun's limb and for the second exposure about 1.3 diameters. The two exposures overlap by about one-eighth of their combined width and the plate was somewhat fogged by extraneous light from too early drawing of the slide. These faults and the incorrect placing of the slit for the two spectra made them unsuitable for the comparison of the coronal spectrum of the polar with the equatorial region.

The presence of the cloud of haze over the sun raises a question as to the extent reflected sunlight contributed to the dark-line spectrum. The direct photographs and also the slit spectra bear evidence of light scattered from the prominences and bright inner corona. The great extension of the calcium and hydrogen prominence lines and their presence over the moon's image is clear

proof that prominence light was widely scattered. Some years ago the late Lord Rayleigh showed how rapidly the power of our atmosphere in scattering sunlight increased as the wave-length was decreased, and previous eclipse spectra have shown the effect with clear skies. Hence our air must have greatly aided the cloud in scattering the violet rays among which the dark lines, in the present case, were observed. It does not seem possible to determine just how great the effect of the haze was, but the presence of the prominence emissions over these spectrograms leaves no doubt that the strictly continuous spectrum of the bright inner corona was likewise spread over that of the outer corona, which would tend to cover up any dark lines it possessed. It is not possible to know to what extent these two opposing effects of the haze balanced each other, and hence we cannot say certainly that the dark lines observed were of true coronal origin although they appear to be such.

Three-prism spectrograph.—The Brashear stellar spectrograph of three prisms, supplied with a camera of nine inches focus, was adjusted for the visual end of the coronal spectrum with the hope of photographing more of the red region than is usually done. I had planned to sensitize a plate to the extreme red, but conditions did not permit my doing that and a Cramer Trichromatic plate was finally used.

The exposure ran from count 2 to 80. The slit was placed east and west. The plate records well the continuous spectrum from the blue down to D_3, where it rapidly weakens owing to the diminished sensitiveness of this emulsion in that region. On the shorter wave-length side of D_3 there are present the usual hydrogen and helium prominence lines and also a number of chromospheric lines. The C line of hydrogen is registered, but no other bright lines are evident on the red side of the D_3 line. The green coronium line λ 5303.0 is strongly impressed upon the plate (Fig. 2), but I have recognized no other true coronal lines.

The green line is long and nearly equally intense on the east and the west of the sun. Prominence and chromospheric lines, however, are much stronger on the east than on the west side, implying that the exposure began a little nearer second contact than

the ending was to third contact. The comparison spectrum was unfortunately accidentally omitted from this plate, but the wavelength of the green line was determined with fair accuracy from measures of it relative to nearby prominence and chromospheric lines, the resulting value being λ 5303.0. Owing to scattering of light the green coronium line, like Hβ and Hγ, is traceable across the moon's image.

The slitless spectrograph.—A battery of three 63° dense flint prisms and a camera of 35-mm aperture and 472-mm focus served as a prismatic camera for investigating the spectrum of the corona with special regard to the green coronium ring. The adjustments were such as to include the spectrum from D_3 to beyond F in the blue, in order to have these helium and hydrogen radiations for comparison with the coronium ring in studying the distribution of this substance around the sun's surface. The prisms were not at minimum for λ 5303 and the green ring is somewhat elliptical. The edges of the prisms were set north and south instead of east and west, the more usual position. This orientation was the more favorable for recording the coronal ring clearly along the east and west limbs where it shows most intensity and detail. In this case the east and west limbs are in the middle of the spectral band and the north and south limbs at its edges, where details are of course confused. On account of the persistent clouds on the last few days and nights I did not get the focus sharply adjusted; and, too, the instrument received a slight jar when the small spectrograph described above was rotated at mid-totality, the two being mounted on the same polar axis. In consequence the objective spectra are not quite sharp, but it seems that nothing of importance was thereby lost.

The solar image on the plate of the prismatic camera had a diameter of 4.3 mm.

Three exposures were made with this instrument on isochromatic plates; one from the signal "go," for three seconds; the second (on the same plate shifted) for seventy seconds; and the third (on a separate plate) for about a second near third contact, for the flash. This last exposure was a little late and owing to the low dispersion is overexposed and probably does not merit any further consideration and description here.

The first of these exposures records the eastern crescents of F and D_3 together with a faint impression of the green coronal crescent. The second, the long exposure, secured a good impression of the green ring and of the irregular, broken rings of hydrogen F and helium D_3. The plate is of course considerably fogged by scattered light and by the light of the bright inner corona. There is sharp increase in the intensity of the continuous spectrum of the corona at its edges corresponding to the north and south limbs of the sun. These margins are strikingly sharp. They of course mask the coronium ring near the north and south points of the sun's disk; but as the ring becomes weak before reaching the edges except in the southwest quadrant, probably not much detail of the ring was thereby covered. However, it is strongly advised in such observations of future eclipses that the observer provide himself with a pair of instruments of this simple type and mount them so that for one the refracting edge of the prism (or grating rulings) is north and south, and for the other east and west. In this way, between the two, he will secure the whole of the sun's limb without masking, and for the most part he will thus also secure the much-to-be-desired duplicate records of the coronium spectrum.

The second exposure, with this instrument, of seventy seconds, began and ended about equally near second and third contacts, respectively, but it ended a very little nearer third contact. This negative shows that the green coronium ring is not uniformly intense around the sun. Its strength is much greater along the east and west limbs, generally, than about the north and south ones, yet its stronger parts do not coincide with the equatorial regions or its fainter parts with the polar regions. It occupies a greater extent of the west limb than it does of the east one, and on that side it extends in considerable strength farther to the south of the equator than it does to the north, in fact it seems traceable to the south pole. On the east limb, on the other hand, the ring stops quite suddenly nearer the equator on the south than on the north. That is, the coronium ring is faint or absent along the sections of the sun's limb occupied by the bristling streamers typical of the polar regions. The spectrogram, compared with direct photographs, shows that the coronium substance was relatively

abundant generally in those regions of the sun's limb from which flowed the great extensions of the corona. While there seems to be this general correspondence in the distribution of the coronium substance and the main features of the corona, the nature of the relationship is a problem for further investigation.

In addition to these main variations in intensity of the coronium ring it has numerous local irregularities, its stronger portions being found in the zones of spots and prominences, generally; yet these do not coincide with individual prominences. The presence on the same plate of the F ring of hydrogen and the D_3 ring of helium, one on either side of the coronium ring, rendered comparison and study easy and definite. It is evident at a glance that the coronium ring is not at all like those of hydrogen and helium. The distribution of coronium thus is different from that of hydrogen and helium, and so no obvious relationship of coronium to these other substances is indicated. It is true only in a general way that the green ring is strong throughout regions including the zones frequented by sun-spots and prominences. The condensations in the coronium ring, none of which are definitely limited, stand in striking contrast to those of hydrogen and helium due to the prominences. The spectrogram shows the green radiation to fade, at first rather rapidly, outward from the sun's surface. While this negative gives no direct evidence on the question whether or not coronium is concerned in the arches over the prominences, it reveals a distribution of coronium favorable to its presence there. The coronium line on the plate of low dispersion, already described, extends outward fully a solar diameter into the corona. It is stronger off the southwest limb than off the southeast one—both of which were similarly included by the slit—thus conforming to the intensity-distribution of the coronium ring shown on the objective-prism plate. Of course we must consider here the effect that scattering of light in the earth's atmosphere has upon the extension of the line. Some assistance on this point is furnished by the negative for the general coronal spectrum, the first to be described of the series. This negative contains a good record of the emissions of hydrogen, helium, calcium, and coronium from the yellow to the ultra-violet, and, as it is without a comparison

spectrum, it is well adapted to comparing the emission lines inside the moon's limb, where scattering is wholly responsible for their presence, with their extensions outward from the same limb into the corona, where the effect is a combination of scattering of light and the distribution of the substances involved. Inside the moon's disk the coronium line is as weak as, or weaker than, the helium line λ 4472 and, of course, weaker than any of the hydrogen lines; but when compared with the same lines at, say, half the solar radius outside the moon's limb, on the west, the coronium line is as strong as the strongest hydrogen line. Lord Rayleigh investigated the scattering of light in the atmosphere and found it to increase inversely as the fourth power of the wave-length. The effect is clearly disclosed by this spectrogram, the coronium line showing less spreading over the moon's image than do the blue and violet lines of hydrogen and helium. But outside the sun's limb over the corona something more than scattering is at work, for the coronium line gains strength relatively to these same H and He lines of shorter wave-length. This seems to imply that some coronium was present high above the sun's surface, as high apparently as the arches over the prominences.

These arches seemed unusually well developed at this eclipse and it is unfortunate that the presence (or absence) in them of coronium could not be cleared up. The presence of the coronium line high above the sun raises the question whether its luminescence may not be, as has been suggested, an electro-magnetic phenomenon similar to that of the aurora: the arches over the prominences have some resemblance to magnetic lines of force. One might further remark upon the fact that coronium and "aurorium" each has its distinguishing radiation in the same general region of the spectrum. Much remains to be learned of coronium and its relationships, at future eclipses.

This objective-prism plate, Figure 3, has further interest in that it shows strips of well-defined, continuous spectrum which run longitudinally through the main spectral background of the inner corona. Three of these are quite distinct and rather definitely limited in width. It appears evident that two of them coincide with the two

strong prominences of the northern hemisphere, one on the east limb, the other on the west. The third is in the southern hemisphere and is somewhat wider and apparently related to some small prominences and an elevated region of the photosphere. Comparison of the maxima of the coronium ring with those of the H and He rings with regard to the strips of intensified continuous spectrum led me to the conclusion that the bands are related to the prominences and photosphere and not in any direct way to the substance of the green ring. In addition to what has been stated, the facts pointing to this conclusion are that the maxima of the ring are not definitely limited, as are the strips of continuous spectrum and there is not that correspondence between the two that would be expected were the ring maxima and the strips due to the same origin. It is noted that where the bands cross the ring it is generally denser, but it seems not more so than would result from the action of the blended light of the two.

It seems to be definitely established that some prominences possess a continuous spectrum, and brief computation shows that a depression of the moon's limb, moderate in depth among those known to exist[1], would have been equivalent to starting the exposure as much as 5 seconds early—or to stopping it that much late—which is ample time, near second or third contact, to expose sections of the photosphere and so introduce continuous spectrum. Observations clearly bear this out, as such lunar depressions have caused sharp strips of continuous spectrum on eclipse spectra in numerous cases. Both the causes mentioned above seem to have been concerned in the strips of continuous spectrum described here for the 1918 eclipse.

To Mrs. Percival Lowell, for her generous provision for the expedition and interest in its success; to Mr. Guy Lowell, the Trustee of the Lowell Observatory, who early proposed the undertaking; to Professor W. W. Campbell, Director of the Lick Observatory, for expert advice on plans and equipment; to Professor

[1] *Astronomische Abhandlungen der Hamburger Sternwarte in Bergedorf*, **3**, No. 1; *Selenographische Koordinaten von Friedrich Hayn.* IV Abhandlung des XXXIII Bandes, K. Sternwarte, Leipzig; *Astronomische Nachrichten*, No. 4615 (**193**, 118, 1912).

John A. Miller, Director of the Sproul Observatory, and Professor W. A. Cogshall, Director of the Kirkwood Observatory, for counsel and the loan of apparatus; to Mr. George R. Agassiz, Boston, Massachusetts, for the loan of a large lens; to the Naval, Flower, and Haverford observatories, for the loan of equipment, and to a number of citizens of Syracuse for their kind interest and assistance before and on eclipse day, it is a pleasure to make grateful acknowledgments.

Lowell Observatory
Flagstaff, Arizona
October 1921

SPECTROPHOTOMETRIC METHODS FOR DETERMINING STELLAR LUMINOSITY[1]

By BERTIL LINDBLAD

ABSTRACT

Two spectrophotometric methods for determining absolute magnitudes.—(1) *Relative intensity of spectral regions on either side of λ 3907.* Using the 10-inch Cooke refractor with 6° objective prism, a series of images of decreasing density was obtained for each star on each plate, making each exposure 0.707 times the preceding, and from these images the exposure ratio ($E=t_2/t_1$) required to give the same density in the two adjacent spectral regions was determined for stars in well-known groups with common proper motions and hence common parallaxes—the Hyades, the Pleiades, the Ursa Major stream, and Praesepe—and also for some stars with known spectroscopic parallaxes, in all, 107 stars of types A and B and 74 stars of later types. The results show that, especially *for stars of type B8 to A3*, there is a definite correlation between E and M, the variation being such that even with the small dispersion here used the probable error of a determination of M for a star within this range of type and of absolute magnitude 0 to 4 is only $0^M.4$. (2) *Relative intensity of cyanogen bands and adjacent spectral regions.* Using the same instrument and method as above, the relative density on either side of λ *3889* was determined for 16 stars of known parallax, and (since the band at λ 4216 though weak is more favorable for the redder stars) the relative density in regions λλ 4144–4184 and λλ 4227–4272 was determined for 78 late stars. The absorption was found to be strongest *for the giants of types G5–K5* and much weaker for dwarfs; in fact all stars with log $E>0.17$ are giants. Figures 3–5 show the variation of log E with M for various types.

Relation of spectra of A and B stars to absolute magnitude.—A study of some spectra made with the 60-inch showed that the increase in the density of the region λλ 3889–3907 with reference to λλ 3907–3935 is caused by a widening of the Hζ line and also of some arc lines of Fe and Si, which may be due to a greater density gradient in the outer layers of the less luminous stars of this type. α Cygni and the faint companion of o_2 Eridani represent extreme types.

Spectral type and luminosity of 137 faint stars of magnitudes 10 to 14, in Messier 11, Messier 13, and Selected Areas 63 and 64, were determined from 10 plates made with a small focal-plane spectrograph attached to the 60-inch. Dwarfs and giants were distinguished by applying the spectrophotometric results reported above.

Spectrophotometric parallax determinations.—The concordance of the results for log E as a function of M, as determined from the various groups of stars, indicates that the assumed parallaxes for the *Hyades, Pleiades, and Praesepe*, $0''.023$, $0''.010$, and $0''.0072$, respectively, are accurate to within 10 per cent. The value found for *Messier 11* is $0''.00025$.

Schwarzschild's constant p for Seed 23 plates was determined for images made with the 10-inch refractor and 6° prism and found to be equal to: 0.97–0.026 ($m-5$), where m is apparent magnitude ranging from 4 to 8. X-ray developer was used.

I. A CRITERION OF ABSOLUTE LUMINOSITY FOR STARS OF SPECTRAL TYPE A

The faint companion of the star o_2 Eridani of apparent magnitude 9.5 and absolute magnitude +10.3 was found by Adams[2] to be

[1] *Contributions from the Mount Wilson Observatory*, No. 228.

[2] *Publications of the Astronomical Society of the Pacific*, **26**, 198, 1914.

of spectral type A0, and, intrinsically, is by far the faintest A-type star known. Some photographs of this object, taken with an objective prism on the 10-inch refractor and with a focal-plane spectrograph in conjunction with the 60-inch reflector, showed some peculiarities in the energy-distribution of the spectrum, when compared, for instance, with the spectrum of Sirius. The most conspicuous feature is the very strong widening of the wings of the hydrogen lines, connected with a decided weakening of the high-numbered members of the Balmer series. On the spectrograms of low dispersion taken here, Hη is the last hydrogen line to be identified with certainty. Seemingly as a consequence of this weakening of the extreme hydrogen lines, the photographs show a faint extension of the continuous spectrum much farther into the ultra-violet than is the case for an ordinary A-type star. A further difference between this star and Sirius was found in the distribution of energy between Hϵ and Hζ, apparently consisting in a considerable decrease of intensity in the region λ 3889–λ 3907 as compared with the region λ 3907–λ 3935. This phenomenon may be caused partly by the expansion of the wings of Hζ, but there was seemingly also some other cause; in particular, a widening of the silicon arc line λ 3905.6 was suspected. This line is well developed in the spectrum of Sirius, though probably it is to some extent a blend with an enhanced line of chromium. In order to see whether the last mentioned phenomenon might be a general criterion of intrinsic brightness for A-type stars, an investigation was begun on the Hyades which was later extended to include other groups of common proper motion.

The instrument used was the 10-inch Cooke refractor with an objective prism of 6°. The scale of the spectra thus obtained, about 340 A per mm in the region of the K line, is probably near the lower limit possible for a successful investigation of the effect. No widening of the spectra was undertaken. Satisfactory images of an A star of photographic magnitude 6.5 were thus obtained in one minute. Seed 23 plates were used, with X-ray developer. Owing to the steep color-curve of the objective for the extreme violet, only a small region of the spectrum can be in good focus for a given setting; that used corresponded to the interval λ 3889–λ 3920.

In order to measure the decrease of intensity at λ 3907 the following process was used. For each object a series of exposures was made on the same plate with exposure-times decreasing in the ratio $\sqrt{2}:1$. Each star thus produced a sequence of images of decreasing photographic density; that is, a scale of images which may be represented by the numbers 0, 1, 2, 3, etc., corresponding to the exposure-times t_0, t_1, t_2, t_3 . . . . , where $t_i : t_{i+1} = \sqrt{2}$. The density in the region λ 3895 to λ 3907 of a certain image, for instance, the first, was compared with that in the region λ 3907 to λ 3925 of the following images. The point in the scale of images for which the density in the second region equaled that of the first region in the reference image, was estimated by interpolation to a tenth of an interval. Suppose the exposure-time corresponding to this "scale-reading" to be t. The logarithm of the ratio $t_0 : t$, denoted by log E, is then a measure of the energy-fall in the spectrum near λ 3907. The successive images define a scale with intervals of logarithmic exposure-ratio equal to 0.15, and the "scale-reading" observed is therefore readily converted into log E. As many estimations as possible were made from each series of images, starting with each of the spectra in succession as a reference image. The process is analogous to that used by Seares[1] in determining the color of a star by the method of exposure-ratios. Distorted images, as well as very dense or very weak images, have not been measured. Whenever possible, measurements were made with the plate in two opposite positions. In some of the measurements, especially of very bright stars, like Sirius, for which sufficiently short exposures could not be made with the accuracy necessary to form a reliable scale, the value of log E was estimated directly from the difference of density between the two regions in the same image. Apart from the influence of possible changes of gradation from one plate to another, the two methods, if checked against each other, should give equivalent results. This follows from Kron's[2] results for the law of photographic density as a function of intensity and exposure-time. If the gradation in a time-scale is constant, a difference in density obtained during the same

[1] *Proceedings of the National Academy of Sciences*, **2**, 521, 1916.

[2] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **22** (No. 67), 1913.

exposure for two different light-sources will always correspond to the same ratio of exposure-times giving the same density, independently of the order of magnitude of the intensities, in this case, of the apparent magnitude of the star.

The wave-length λ 3907, separating the two regions, is easily identified with sufficient accuracy from its position relative to the lines Hζ, Hε, and K. The abrupt fall of intensity near this wave-length for low-luminosity A-type stars was confirmed by these observations and also on spectrograms of higher dispersion, as will be seen later. With the low dispersion used, the rapid change in intensity leaves the density fairly uniform in the two regions, λ 3895–λ 3907 and λ 3907–λ 3925. In fact, the success of the process depends on this circumstance. On the other hand, the smallness of the dispersion entails relatively large accidental errors, thus necessitating the use of a rather large number of images in order to get a reliable mean result.

Because of the juxtaposition of the two regions in the spectrum, it is evident that the influence of slight changes in the sensibility-curve of the plate, as well as of a variation in zenith-distance within reasonable limits from one object to another, will be negligible. The same will hold for a difference of gradation for the two regions. Further, if we use the law of density in Schwarzschild's form, $S=\phi\,(It^p)$, we may assume the value of p to be the same for the two regions. Suppose I_1 and I_2 to be the acting intensities for the first and second regions, respectively, and suppose S_1, t_1, S_2, t_2, to be corresponding values of density and exposure-time for the two regions. Then for equal densities we have $S_1=S_2$, and

$$I_1 t_1^p = I_2 t_2^p$$

$$\log \frac{I_2}{I_1} = p \log \frac{t_1}{t_2} = p \log E\,.$$

If we use the same kind of plates and the same developer, we have reason to assume p to be tolerably constant, though experience shows that large deviations may occur.

Since we may certainly disregard any difference in gradation for the two regions, there cannot, according to Kron's results, be any change of log E with the density of the images, a result which

seems to be confirmed by the measures. This follows from the parallelism of the curves log It against log I for different degrees of density in Kron's diagrams. Two values of log I giving the same density will demand the same difference of log t for all values of the density. There may be, however, a slight change of log E with the absolute value of the acting intensity, that is, with the apparent magnitude of the star, due to the bending of the curves mentioned, corresponding to a change of the exponent p with the intensity.

The method described here admits a determination of Schwarzschild's constant p for a certain intensity, as soon as we have on a single plate the series of images of two different stars whose magnitudes are accurately known. The method of measurement is simply to interpolate the density of the successive images of one star into the scale defined by the images of the second star. In this way p has been determined from the relation $p=\frac{\Delta \log I}{\Delta \log t}$ for nearly all of the plates used here; in some cases p was determined for very different intensities on the same plate. The results seem to confirm Kron's conclusions. There is a decided decrease of p with the apparent magnitude. The mean from five plates is

$$\frac{\Delta p}{\Delta m}=-0.026\pm0.002 \text{ (P.E.)},$$

for photographic magnitude 6.5.

Assuming a linear relation between p and m to hold for the variation of intensity met with in this investigation, we have as a mean for the first seventeen plates measured $p=0.97-0.026\,(m-5)$. Individual plates were assumed to differ from this equation by a constant ϵ, which in so far as possible was determined for each plate. The probable deviation in p from the formula for one plate, computed as a probable error for the seventeen plates mentioned, was found to be ±0.03. A few Seed 23 plates of a later emulsion give, in the mean, lower values of p. One plate of the Pleiades was discarded by reason of an abnormally low value of p. The values of log E were reduced, as far as possible, to $p=0.97$. As a rule the corrections were small.

From stars measured on more than two plates it appears that the probable error of log E from a series of four measured images on a single plate is ± 0.025. The residuals of different plates indicate a probable systematic error of about ± 0.02. A considerable part of the probable error in log E for one plate must therefore be due to causes common to all the stars on the plate taken in the same exposure. For the greater part these are probably connected with irregularities in the images caused by imperfect guiding. From the foregoing discussion it seems unlikely that varying quality of the plates contributes appreciably. In computing the final values of log E, weights were therefore assigned equal to the number of images measured on the respective plates.

Attention was given first to members of well-known groups of stars with common proper motions, the Hyades, the Pleiades, the Ursa Major stream, and Praesepe, for which values of the parallaxes have been derived from the stream-motion or other plausible considerations. The large number of early-type stars in these clusters makes it much easier to establish the relation of spectral intensity to absolute magnitude than is the case for later types. Tables I–IV contain the results for stars of these groups; in addition, Table V gives results for some stars of types B5 to B9 of Kapteyn's groups in Orion and Scorpius, and for the two stars α Cygni and Comp. o_2 Eridani, which may be considered as representing extreme limits for the luminosities of A-type stars.

For the Hyades the parallax used for computing the luminosities is that of Kapteyn-Küstner,[1] $\pi = 0''.023$. The Ursa Major stars are given according to Bottlinger.[2] Thirteen of the fifteen A-type stars which are certainly members of the group have been observed for this investigation. The absolute magnitudes of ϵ and ζ_1 Ursae Majoris, spectroscopic binaries with composite spectra of nearly equal intensities, have been increased by $+0^M.75$ in order to reduce to the absolute magnitude of each component. For the Pleiades a list of probable members brighter than $9^M.5$ photographic has kindly been communicated by Dr. Trumpler of the Lick Observatory. The value of the parallax adopted is Trumpler's[3]

[1] *Publications of the Astronomical Laboratory at Groningen*, No. 23, 1909.

[2] *Astronomische Nachrichten*, **198**, 153, 1914.

[3] *Publications of the Astronomical Society of the Pacific*, **33**, 214, 1921.

result, $\pi = 0''.010$, derived by means of the orbital motion of ten visual binaries belonging to the group. Some stars of the list are not measurable on the plates because of overlapping images. The maximum exposures for the four plates used were 30 minutes, 10 minutes, 2 minutes, and 30 seconds, respectively. For the Praesepe

TABLE I

HYADES, $\pi = 0''.023$

STAR		VIS. MAG.	SPECTRUM	LOG E	N	n	M_π	M_s	ΔM	Δ LOG E
Gron. 23	B.D.									
—	+15°632.....	3.62	Fo	0.14	1	2	0.43			
—	18 640.....	3.63	G9	0.09	1	8	0.44			
28	15 612.....	3.86	G7	0.05	3	17	0.67			
79	17 712.....	3.93	G9	0.05	3	27	0.74			
224	15 631.....	4.04	G9	0.05	4	26	0.85			
143	17 719.....	4.24	A2	0.15	3	18	1.05	0.8	+0.3	−2
170	15 625.....	4.60	A5	0.17	4	27	1.41	1.1	+0.3	−2
315	14 720.....	4.75	A5	0.19	3	15	1.56			
107	17 714.....	4.84	A5	0.19	3	28	1.65			
385	15 666.....	4.85	A3	0.25	2	12	1.66	3.6	−1.9	+5
251	15 637.....	4.84	A5	0.22	2	9	1.65			
38	14 682.....	5.27	Fo	0.19	3	23	2.08			
255	15 639.....	5.49	A5	0.20	3	16	2.30			
34	13 663.....	5.59	Fo	0.19	3	13	2.40			
87	16 586.....	5.68	A2	0.22	3	18	2.49	1.9	+0.6	−2
245	15 636.....	5.70	Fo	0.23	2	11	2.51			
64	13 668.....	5.76	A3	0.25	3	14	2.57	3.6	−1.0	+2
371	15 661.....	5.80	Fo	0.19	2	10	2.61			
222	14 702.....	5.97	Fo	0.19	3	10	2.78			
282	15 645.....	6.04	Fo	0.17	3	16	2.85			
44	13 665.....	6.14	F2	0.11	3	12	2.85			
1	15 603.....	6.35	F5	0.15	2	6	3.16			
150	+15 621.....	6.39	F8	0.12	4	15	3.20			

cluster Kohlschütter's[1] value $\pi = 0''.0072$ was adopted. This was derived by a comparison of the relations between apparent magnitude and spectrum for the early-type stars in the Hyades and Praesepe. This value is at least consistent since it gives for the four bright K stars of Praesepe a mean absolute magnitude nearly equal to that of the corresponding stars in the Hyades. For one of the Praesepe stars mentioned, B.D.+20°2166, Mr. Adams has kindly secured a spectrogram, from which he estimates the absolute

[1] *Astronomische Nachrichten*, **211**, 289, 1920.

magnitude to be −0.4. A result of the same order is indicated by the strong absorption in the "cyanogen" bands shown by a spectrogram of the same star, taken by the writer with the Crossley reflector, which places the luminosity near that of β Geminorum ($M = +1.7$). The value $M = +0.69$ found with $\pi = 0''.0072$ is in fairly good agreement with these results. The stars from Kapteyn's lists have been combined into groups having the same spectral type and nearly the same values of log E. The absolute magnitude

TABLE II

URSA MAJOR STREAM

Star	Vis. Mag.	Spectrum	log E	N	n	M_π	M_s	ΔM	$\Delta \log E$
α Cor. Bor.	2.31	A0	0.12	2	19	0.37	0.4	0.0	0
ϵ Ursae Maj.	1.68	A0p	0.15	2	10	0.55	0.8	−0.2	+2
β Serpentis	3.74	A2	0.13	2	13	0.56	0.5	+0.1	0
γ Ursae Maj.	2.54	A0	0.14	3	22	0.60	0.6	0.0	0
78 Virginis	4.93	A2p	0.13	2	12	0.66	0.5	+0.2	−1
β Ursae Maj.	2.44	A2	0.15	3	11	0.80	0.8	0.0	0
α Canis Maj.	−1.58	A0	0.20	2	14	1.32	1.5	−0.2	−1
ζ_1 Ursae Maj.	2.40	A0p	0.20	2	8	1.44	1.5	−0.1	+1
76 Cygni	6.05	A0	0.19	1	6	1.62	1.4	+0.2	−1
ξ Eridani	5.23	A2	0.20	1	9	1.63	1.5	+0.1	0
δ Ursae Maj.	3.44	A2	0.21	4	29	1.71	1.7	0.0	0
g Ursae Maj.	4.02	A5	0.18	3	18	2.14	.....	.....	.....
ζ_2 Ursae Maj.	3.96	A0	0.23	1	4	2.25	2.2	0.0	0

was assumed to be the same within each group, and was calculated from the means of Kapteyn's values of the parallaxes.

The fourth column of the tables contains the final values of log E, corrected to $p = 0.97$; the fifth and sixth give respectively the number of plates and the total number of images on which these values are based. The seventh column gives the absolute magnitude M_π, derived from apparent magnitude and parallax by the relation $M = m + 5 + 5 \log \pi$. The eighth column gives the absolute magnitude M_s, read by means of log E from the curves in Figure 1. The ninth and tenth columns give the residuals in M and log E from these curves.

The visual apparent magnitudes for the Hyades, the Ursa Major stars, and for Kapteyn's stars are from the Draper catalogue.

TABLE III

PLEIADES, $\pi = 0''.010$

STAR		VIS. MAG.	SPECTRUM	LOG E	N	n	M_π	M_s	ΔM	Δ LOG E
Bessel	B.D.	Distance from Alcyone $\gtrless 1°$								
η	$+23°541$.....	2.94	B5p	0.06	1	4	−2.06	.....	.....	.....
f	23 557.....	3.73	B8	0.06	2	8	−1.27	.....	.....	.....
b	23 507	3.76	B5p	0.11	3	12	−1.24	.....	.....	.....
c	23 516	3.93	B5	0.08	2	7	−1.07	.....	.....	.....
ϵ	23 547.....	4.34	B5	0.09	2	11	−0.66	.....	.....	.....
d	23 522.....	4.28	B5	0.06	1	3	−0.72	.....	.....	.....
h	23 558.....	5.15	B8p	0.11	3	10	+0.15	0.2	0.0	+1
g	23 505.....	5.53	B5	0.11	1	4	0.53	0.2	+0.3	−2
28	22 563.....	5.54	B8	0.16	2	9	0.54	0.9	−0.4	+3
m	24 546.....	5.68	B8	0.09	2	11	0.68	.....	.....	−5
k	24 553.....	5.83	B8	0.12	2	8	0.83	0.4	+0.4	−3
34	23 563.....	6.23	B8	0.19	3	12	1.23	1.4	−0.2	+1
32	23 561.....	6.66	B9	0.17	1	4	1.66	1.1	+0.6	−4
38	23 569.....	6.82	B9	0.24	3	15	1.82	2.5	−0.7	+3
12	24 562.....	6.88	B9	0.22	3	12	1.88	2.0	−0.1	0
24	23 540.....	6.93	B9	0.22	3	11	1.93	2.0	−0.1	+1
40	23 570.....	6.98	A0	0.23	3	17	1.98	2.2	−0.2	+1
29	23 553.....	6.99	A0	0.22	3	14	1.99	2.0	0.0	0
19	23 537.....	7.04	A0	0.24	3	16	2.04	2.5	−0.5	+2
22	23 538.....	7.19	B9	0.25	3	12	2.19	2.8	−0.6	+2
10	23 523.....	7.37	A0	0.24	3	15	2.37	2.5	−0.1	0
37	23 567.....	7.44	A0	0.22	3	13	2.44	2.0	+0.4	−2
39	24 578.....	7.46	A2	0.23	3	11	2.46	2.2	+0.3	−1
31	23 560	7.54	A0	0.28	1	3	2.54	3.6	−1.1	+4
20	24 566.....	7.68	A2	0.26	2	6	2.68	3.1	−0.4	+1
	24 540.....	7.79	A3	0.26	2	7	2.79	4.0	−1.2	+3
23	23 539.....	7.81	A2	0.24	2	5	2.81	2.5	+0.3	−1
7	23 517.....	7.87	A2	0.27	2	8	2.87	3.3	−0.4	+2
33	23 562	8.01	A2	0.23	2	5	3.01	2.2	+0.8	−3
1	23 510.....	8.06	A2	0.27	3	6	3.06	3.3	−0.2	+1
	23 496.....	8.07	A5	0.21	2	7	3.07	.....	.....	.....
	23 524	8.13	A2	0.29	2	7	3.13	3.9	−0.8	+3
30	23 559.....	8.14	A3	0.25	2	6	3.14	3.6	−0.5	+1
	23 504.....	8.22	A5	0.22	2	6	3.22	.....	.....	.....
	22 573.....	8.24	A3	0.23	2	6	3.24	2.6	+0.6	−1
	23 495	8.29	A5	0.19	2	9	3.29	.....	.....	.....
13	23 528.....	8.46	A5	0.24	2	5	3.46	.....	.....	.....
	23 508.....	8.51	A3	0.24	2	4	3.51	3.1	+0.4	−1
	22 565.....	8.53	A5	0.21	2	6	3.53	.....	.....	.....
	$+24$ 563.....	8.59	A5	0.20	2	5	3.59	.....	.....	.....

TABLE III—*Continued*

Star		Vis. Mag.	Spectrum	log E	N	n	M_π	M_s	ΔM	Δ log E
Bessel	B.D.	Distance from Alcyone 1° to 3°								
	+21°535....	6.0	B9	0.15	3	17	1.0	0.8	+0.2	−1
	22 523....	6.7	A0	0.17	3	18	1.7	1.1	+0.6	−4
	22 545....	6.9	A0	0.22	3	17	1.9	2.0	−0.1	+1
	22 572....	6.9	B9	0.22	3	16	1.9	2.0	−0.1	0
	24 527....	7.1	A0	0.21	3	18	2.1	1.7	+0.4	−2
	21 492 ...	7.4	A0	0.20	3	16	2.4	1.6	+0.8	−4
	24 537....	7.6	A3	0.23	3	14	2.6	2.6	0.0	0
	20 624....	7.7	A2	0.20	3	9	2.7	1.6	+1.1	−5
	22 569....	7.8	A2	0.29	3	13	2.8	3.9	−1.1	+4
	25 631....	7.8	A2	0.24	3	12	2.8	2.5	+0.3	−1
	22 544....	8.0	A2	0.22	3	10	3.0	2.0	+1.0	−4
	24 554....	8.2	A3	0.26	2	8	3.2	4.0	−0.8	+2
	24 568 ...	8.4	A3	0.21	2	8	3.4	1.9	+1.5	−4
	22 556....	8.5	A2	0.27	2	7	3.5	3.4	+0.1	−1
	21 530....	8.3	Fo	0.13	2	6	3.3	.....	.....	.....
	24 528....	8.9	A5	0.22	2	4	3.9	.....	.....	.....
	+25 615....	8.9	A3	0.25	2	4	3.9	3.6	+0.3	−1

TABLE IV

PRAESEPE, $\pi = 0''.0072$

Star	Vis. Mag.	Spectrum	log E	N	n	M_π	M_s	ΔM	Δ log E
B.D.									
+20°2171.....	6.23	A2	0.17	2	14	0.52	1.1	−0.6	+4
20 2149.....	6.39	Fo	0.15	2	10	0.68	.....	.....	.....
20 2166.....	6.40	G5	0.05	2	2	0.69	.....	.....	.....
20 2158.....	6.48	K0	0.08	2	5	0.77	.....	.....	.....
20 2159.....	6.48	A0	0.11	2	6	0.77	0.2	+0.6	−4
19 2069.....	6.54	Fo	0.14	2	9	0.83	.....	.....	.....
19 2053.....	6.58	A5	0.13	2	13	0.87	.....	.....	.....
20 2175.....	6.60	A3	0.17	2	10	0.89	1.1	−0.2	+1
20 2172.....	6.77	A5	0.19	2	9	1.06	.....	.....	.....
20 2185.....	6.98	K0	0.10	2	3	1.37	.....	.....	.....
20 2143.....	7.09	Fo	0.20	2	5	1.38	.....	.....	.....
20 2152.....	7.26	A0	0.16	2	8	1.55	0.9	+0.7	−4
20 2163.....	7.41	A0	0.24	1	4	1.70	2.5	−0.8	+3
20 2153.....	7.52	A0	0.18	2	6	1.81	1.2	+0.6	−3
19 2064.....	7.54	A3	0.17	2	7	1.83	1.1	+0.7	−4
20 2148.....	7.57	A3	0.21	2	6	1.86	1.9	+0.0	0
20 2132.....	7.72	A0	0.26	2	6	2.01	3.1	−1.1	+4
20 2165.....	7.74	A0	0.20	2	6	2.03	1.6	+0.4	−2
19 2078.....	7.77	A3	0.21	2	5	2.06	1.9	+0.2	−1
+19 2084.....	7.88	A5	0.20	2	6	2.17	.....	.....	.....

Those of the Pleiades are by Miss Parsons,[1] or by Müller and Kempf with the reduction −0.30 to the Harvard scale. For a few Pleiades stars Münch's photographic magnitudes have been used, reduced to the visual scale by means of color-indices inferred from the spectral type, which is not far from Ao. The first part of Table III contains members of the group within 1°; the second,

TABLE V

KAPTEYN STARS, COMP. o_2 ERIDANI, α CYGNI

Star	Vis. Mag.	Spectrum	log E	Mean	N	n	M_π	M_s	ΔM	$\Delta \log E$
Boss 4018.....	4.11	B9	0.11	.....	1	7	.....	....	.....	.....
1438.....	4.92	B9	0.10	.....	2	12	.....	....	.....	.....
1455.....	5.57	B9	0.11	0.12	3	19	0.01	0.4	−0.4	+2
1448.....	6.61	B9	0.16	.....	3	11	.....	....	.....	.....
Boss 1944.....	3.09	B8	0.08	.....	1	5	.....	....	.....	.....
4465.....	4.65	B8	0.08	.....	1	5	.....	....	.....	.....
2071.....	5.11	B8	0.08	.....	1	5	.....	....	.....	.....
4058.....	5.41	B8	0.07	0.08	1	5	−0.16	....	.....	.....
4006.....	5.61	B8	0.07	.....	1	6	.....	....	.....	.....
4573.....	5.79	B8	0.06	.....	1	5	.....	....	.....	.....
1788.....	5.88	B8	0.08	.....	1	6	.....	....	.....	.....
Boss 1262.....	3.68	B5	0.04	0.05	1	8	.....	....	.....	.....
926.....	5.25	B5	0.06	.....	1	7	−1.27	....	.....	.....
Comp. o_2 Eridani	9.5	B9	0.31	.....	2	2	10.3	....	.....	.....
α Cygni.....	1.33	A2*p*	0.08	.....	1	8	.....	....	.....	.....

between 1° and 3° from Alcyone. For the latter stars the photographic magnitudes derived by Trumpler have been reduced to the visual scale in the same way as those by Münch. For Praesepe the photographic magnitudes by Kohlschütter on the Göttingen system were similarly reduced to visual magnitudes. The spectra are given according to Harvard with the exceptoin that for Pleiades stars fainter than $7^M.5$ the spectral types have been revised on the basis of the strength of the K line.

Table VI contains the results of measurements on later-type stars from the list of spectroscopic parallaxes by Adams, Joy, Strömberg, and Burwell, made in connection with an investigation of the "cyanogen" absorption beyond λ 3889. The means were

[1] *Astrophysical Journal*, **47**, 38, 1918.

taken for limited intervals of spectral type and absolute magnitude; N is the number of stars in each group. No correction was made for a variation of Schwarzschild's constant, and the measurements for the individual stars are not as accurate as for the A-type stars in the preceding tables. In fact, with the decrease in intensity of Hζ and the general strengthening of the metallic lines in later spectral types the region λ 3895–λ 3907 has no longer the same uniform appearance as for the early types. The results in the table show, however, that the effect still persists to some extent in the later types, in spite of the vanishing of Hζ.

TABLE VI

MEAN RESULTS FOR STARS WITH DETERMINED SPECTROSCOPIC PARALLAXES

Spectral Range	Mean Type	N	Mean Abs. Mag.	Mean log E
F5p–G3p	G0p	4	−2.5	0.05
F4 –G8	G1	3	−1.7	0.04
A8 –F9	F4	3	0.3	0.09
F1 –G1	F7	15	3.9	0.15
G1 –K2	G8	12	0.9	0.05
G4 –K2	G9	6	5.7	0.16
K3 –Ma	K5	7	0.6	0.12
K3 –K7	K5	3	7.3	0.24

Figures 1 and 2 illustrate the result in the tables, log E being plotted against M_π, with different symbols for the different groups. α Cygni and Comp. o_2 Eridani are represented by arrows. The absolute magnitude of α Cygni can only be estimated qualitatively as very bright from exceedingly small proper motion. Comp. o_2 Eridani with $M = 10.3$ falls below the margin of the diagram. Figure 1 gives the results for spectral types B5–A3; Figure 2 for types rom A5 on. From B8 to A2 there is no certain evidence of a change in the curve; but the A3 curve for lower luminosities evidently falls below the B8–A2 curve, and forms a transition to the A5 curve in Figure 2. The degeneration of the curves for types A5–F is clearly a consequence of the decreasing intensity of Hζ with advancing type.

It is evident from these results that the increase of log E with decreasing luminosity can be used for determining absolute magni-

tudes for stars of spectral types B8–A3, but that the curves for later types cannot give very trustworthy results. The curves further show that for very bright stars log E approaches a limiting value representing a very smooth run of the energy-curve between

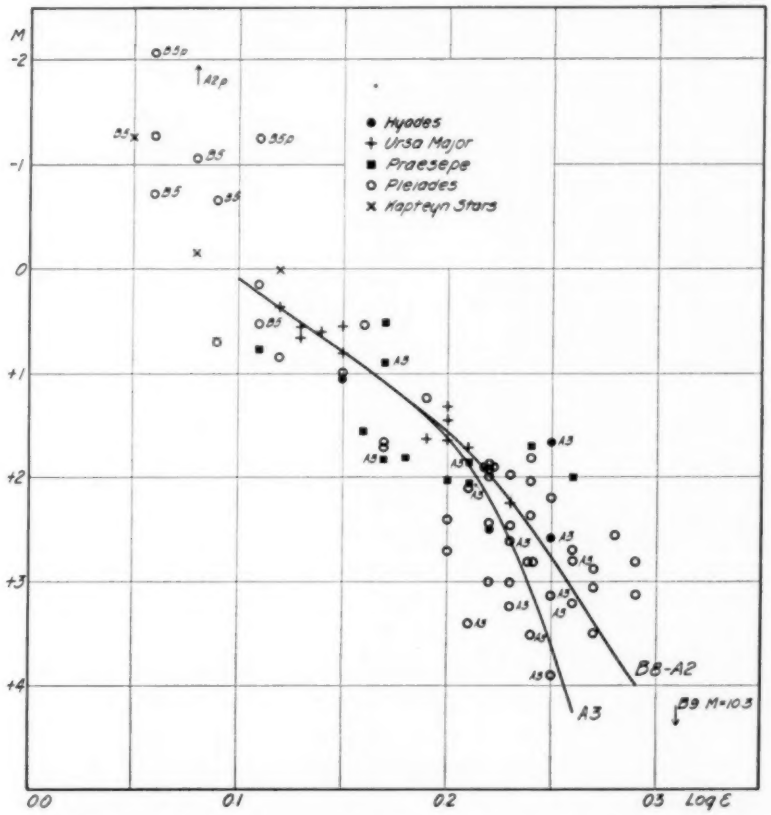

FIG. 1.—Logarithmic exposure-ratio for the regions λλ 3895–3907, 3907–3925, and absolute magnitude. Spectral types B5–A3.

Hζ and K, a consequence of the extreme narrowness of all absorption lines. This phenomenon seems to be more or less independent of type, α Cygni, A2p, falling in with the B5 stars of the Pleiades. The Cepheids and Pseudo-Cepheids and the very bright F4–G8 stars in Table VI also fall near the same point of the diagram. The steepness of the curve for stars brighter than $M = 0$ is such that only

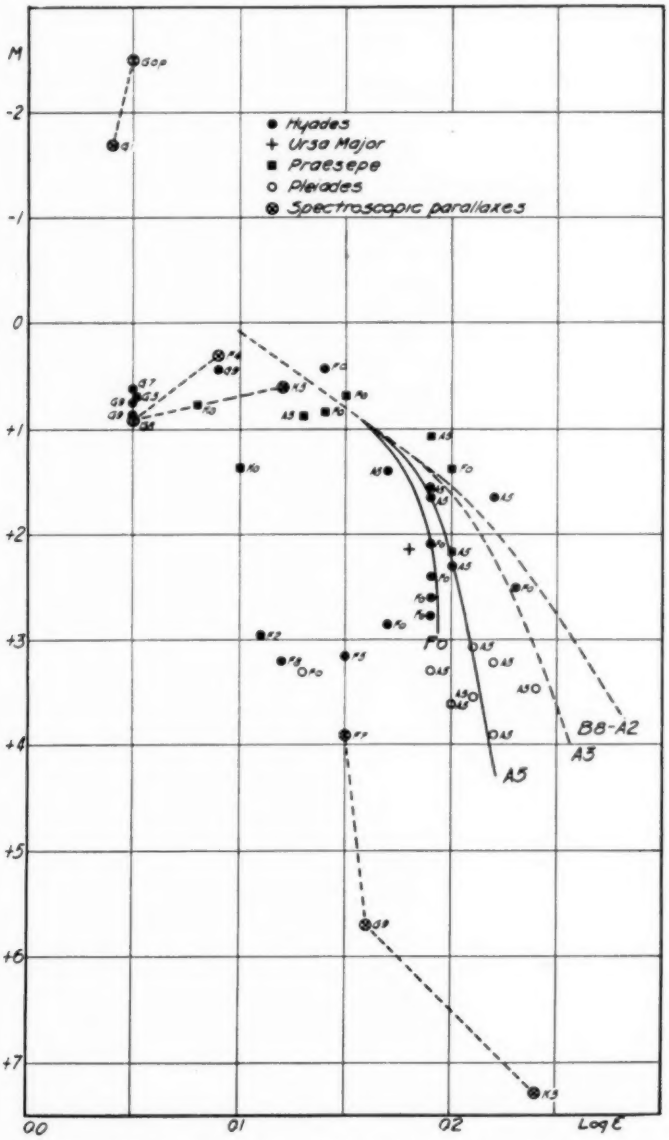

FIG. 2.—Logarithmic exposure-ratio for the regions λλ 3895–3907, 3907–3925, and absolute magnitude. Spectral types later than A5.

qualitative estimates of the luminosity can be made from values of log E less than 0.10.

To form the B8–A2 curve, means of log E for certain intervals of M were taken for both Ursa Major and Pleiades stars. Between $M=0$ and $M=+2$ the curve was drawn to represent the Ursa Major stars. A comparison with the eleven Pleiades stars in this region observed on three plates gave a systematic difference of +0.002 in log E, which is negligible. The remaining parts of the curves were then drawn in accordance with the results for the Pleiades, attention being given also to the Hyades and Praesepe stars in forming the A3 and A5 curves. The co-ordinates of the curves in Figure 1 then run as follows:

	B8–A2				
log E	0.10	0.15	0.20	0.25	0.28
M	+0.08	0.80	1.55	2.77	3.68
	A3				
log E	0.10	0.15	0.20	0.22	0.25
M	+0.08	0.80	1.62	2.20	3.60

The F0 curve in Figure 2 is drawn mainly from stars in the Hyades.

The absolute magnitudes corresponding to log E, M_s, have been read from these curves for all B8–A3 stars having log $E>0.10$; the two A5 stars in Figure 2 with log $E<0.18$ are included.

Errors in the assumed values of the parallaxes evidently combine with residual systematic errors in the measurements. From the discussion of systematic errors it seems that the reliability of any correction to the parallax that might result from the mean residuals for a group will probably depend more on the number of images measured for the individual stars than on the number of stars. The absolute magnitudes of the Ursa Major stars derived from stream-motion are probably but little affected by systematic errors. This likely is also true of the values of log E, because these stars were observed individually on a rather large number of different plates. As there seems to be no appreciable systematic difference

between the Ursa Major stars and the Pleiades we may conclude that Trumpler's value of the parallax, $\pi = 0''.010$, is very good. With allowance for a possible systematic error of about 0.01 in log E the error may amount to about 10 per cent, or $\pm 0''.001$. For the Hyades and Praesepe we can determine the parallax which causes the systematic difference in log E to vanish. For the Hyades we find $\pi = 0''.023$, the value initially adopted; and for Praesepe, $\pi = 0''.0068$, differing by only $0''.0004$ from Kohlschütter's value. An uncertainty of 10 per cent is probably to be admitted for both these values. The Kapteyn stars of types B8 and B9 seem to agree with the curve as well as can be expected.

The residuals ΔM for the individual stars are seen to be very small for the Ursa Major stars, but are rather large for some of the A3 stars of low luminosity in the other groups. This is to be expected from the steepness of the curve. In all, the residuals for seventy-one stars give a probable deviation of $\pm 0^{M}.40$, which is comparable with the probable error of the spectroscopic method for late-type stars. With weights equal to the number of images, n, the probable deviation in log E from the curves is found to be ± 0.026 for a value based on four measured images. This value is to be compared with ± 0.025 previously derived from the residuals in log E from the mean for different plates. The latter value, it was concluded, was produced to a considerable extent by systematic errors common to a large number of stars in a certain group. In the new value, on the other hand, the systematic errors are combined with errors in the parallaxes, and may be partly eliminated in drawing the curves. The close agreement of the two results suggests, however, that the correlation between the energy distribution around λ 3907 and the absolute magnitude is close.

An investigation of spectra of higher dispersion for some selected A stars of different luminosities, photographed with a focal-plane spectrograph in the Newtonian focus of the 60-inch reflector, and with the Cassegrain spectrograph and an 18-inch camera, and also with a slitless quartz spectrograph used in conjunction with the Crossley reflector of the Lick Observatory, throws some light on the question of the physical cause of the effect investigated here. Undoubtedly the increase of log E with decreas-

ing luminosity is to a considerable degree caused by a widening of the wings of the Hζ line, the extreme limits being represented by very bright stars such as α Cygni and α Leonis with narrow hydrogen lines and the very faint companion of o_2 Eridani, which has exceedingly wide lines. But there seems also to be evidence of another cause. It appears as though certain arc lines of iron and silicon in the region λ 3889–λ 3907 increase appreciably in width and intensity with decreasing luminosity. The lines suspected are λλ 3895.8, 3899.9, 3903.1, 3906.6 of iron, and λ 3905.6 of silicon. With the exception of λλ 3903.1 and 3905.6, these lines are classified by King[1] as belonging to Class IB. The exceptions are in classes II and III, respectively. A widening of these lines with decreasing luminosity seems to contribute perceptibly to the depression of the intensity in the region Hζ–λ 3907. There seems to be a similar effect, though much weaker, in the region λ 3914–λ 3923, including the two iron lines λλ 3920.4 and 3923.1 of Class IB. On the whole, in stellar spectra the region between λ 3907 and K is certainly affected by heavy arc lines to a much smaller degree than the region Hζ–λ 3907.

In some spectra of Sirius photographed by Nicholson with the Snow telescope, with a dispersion of 3 A per mm, the lines mentioned between λλ 3889 and 3907 are plainly visible, except the iron line λ 3906.6. The line λ 3905.6 is relatively strong, of about the same strength as the enhanced titanium lines λλ 3900.7 and 3913.7. We probably have here a blend of the silicon line with an enhanced line of chromium, but an indication that the blend is to a considerable part due to silicon may be found in the circumstance that the Kensington observers[2] state that it is somewhat weaker in the spectrum of ε Ursae Majoris than in the spectrum of Sirius, in spite of the fact that of thirty enhanced chromium lines all are found stronger in the former star, except one which is given as of the same intensity in both spectra. In the case of α Lyrae and α Coronae, which are stars of Ao type more luminous than Sirius, λ 3905.6 is stil to be seen in the spectrograms taken with the Cassegrain

[1] *Mt. Wilson Contr.*, No. 66; *Astrophysical Journal*, **37**, 239, 1913; *Publications of the Astronomical Society of the Pacific*, **33**, 106, 1921.

[2] Solar Physics Committee, *Chemical Origin of Various Lines in Solar and Stellar Spectra*, p. 49, 1910.

spectrograph; of the iron lines only λ 3903.1 can be traced with certainty. On the other hand, in ζ_2 Ursae Majoris with $M = +2.25$, λ 3903.1 is strong, and λλ 3895.8 and 3899.9 are clearly visible; in the position of λ 3906.6 there is a very broad line seemingly including λ 3905.6 as a blend. The prominent arc lines of iron between Hζ and Hη are very broad and well marked, probably affecting to a perceptible extent the integrated intensity of the region between these two lines, at least from λ 3860 to Hη. To the red of λ 3907 no effects of similar importance for the variation of the integrated intensity of wide patches of the continuous spectrum can be seen, though the spectrum is evidently rich in lines. The K line, which indicates that the spectral type lies between Ao and A2, is weaker and sharper than in the spectrum of β Serpentis, type A2, $M = 0.56$, in which the iron arc lines are faint. The visibility of the broad arc lines is therefore not likely to be interpreted as due to an increase in spectral type. In addition to these comparisons made with the Cassegrain spectrograph, the slitless spectrograms taken with the quartz spectrograph on the Crossley reflector, and the low dispersion slit spectra obtained with the 60-inch reflector, show an abrupt fall of intensity near λ 3907 for ζ_2 Ursae Majoris, as well as for Nos. 10, 20, and 31 of the Pleiades, types Ao, A2, Ao, $M = +2.37$, 2.68, 2.54, respectively, for No. 64 of the Hyades, A3, $M = +2.57$, and the Praesepe star B.D.$+20°2165$, Ao, $M = +2.03$. The difference relative to the brighter Pleiades and Ursa Major stars is plainly recognizable. A very marked depression near λ 3906 is shown on a spectrogram of Comp. o_2 Eridani taken with the quartz spectrograph. From the appearance of the K line this star ought probably to be classified a little earlier than Ao.

The effect in question persists for later types, as shown by Table VI and is confirmed by results from spectra of other kinds. The energy-fall in a slitless spectrogram at λ 3907 is much more pronounced for a dwarf than for a giant. This fact confirms our results just derived, because the importance of the hydrogen line Hζ for the effect must be negligible in these types. On the other hand, the degeneration of the effect toward type F shows

that Hζ probably causes a large part of the phenomenon in the A type.

The facts dealt with here are probably best understood from the standpoint of an increased density-gradient in the outer layers of a star of certain type with decreasing luminosity, which follows because the change in luminosity is in all probability due chiefly to a change of the mean density. For a layer of certain temperature the density will be appreciably greater in a star of low luminosity than in a more luminous one, a condition which may be thought to favor the strength and width of the arc and flame lines.

The eclipsing binaries offer an opportunity to derive a relation between the effect at λ 3907 and the mean density of the star. From some preliminary results obtained here it has been found that the energy-fall near λ 3907 is strong for RZ Cassiopeiae and RR Draconis, whose brighter components have especially high mean densities, and seems to be much smoother for the low-density stars examined. For RZ Cassiopeiae a value of M between $+2$ and $+3$ inferred from the spectrum is in fair agreement with Dugan's[1] results for the dimensions of the brighter component of this star.

II. THE "CYANOGEN" ABSORPTION IN SPECTRA OF TYPES G–M

Miss Maury[2] makes the statement in her classification of stellar spectra that the sudden decrease of general intensity in the spectrum from λ 3889 toward the ultra-violet found in types XIV and XV, as well as the degree of absorption in the regions λ 4055–λ 4078 and λ 4144–λ 4216, is developed to a different degree for different stars within these types. The variation of intensity in these regions is further thought to be correlated to some extent with the strength of the lines $\lambda\lambda$ 4215.7, 4227, and the compound line λ 4076.8–λ 4077.9. Kapteyn[3] showed that stars with heavy absorption in the violet and ultra-violet regions mentioned have on the average much smaller proper motions than stars with comparatively slight

[1] *Contributions from the Princeton Observatory*, No. 4, 1916.

[2] *Harvard Annals*, **28**, 38, 1897.

[3] *Mt. Wilson Contr.*, No. 31; *Astrophysical Journal*, **29**, 46, 1909.

absorption. This result was thought by him to bear upon the problem of selective absorption in inter-stellar space. In more recent years, however, the influence of differences in the absolute magnitudes of the stars on the appearance of their spectra has been recognized, especially by Adams and Kohlschütter in the development of the spectroscopic method for deriving stellar parallaxes. In this method the lines λ 4216 and λ 4078 are two of the most important variable lines. That the energy-fall beyond λ 3889 is intimately connected with the spectroscopically determined absolute magnitude was found by the writer by using very short objective-grating spectra.[1] The color-effect of the absolute magnitude was found to be much more strongly developed at the inner limit of the spectra in the "minimum wave-length" than in the "effective wave-length" measured by bisecting the images. It was then suspected that the effect could be due to an increase of absorption with stellar luminosity in the heavily winged lines of iron in the region λ 3870–λ 3890.

It has been found recently, however, that the effect is without doubt due to a variation in strength with stellar luminosity in the strong "cyanogen" band with its first head at λ 3883. On spectra of stars of different types and luminosities photographed with a slitless quartz spectrograph[2] on the Crossley reflector of the Lick Observatory three similarly behaving absorption regions were found to coincide with the three "cyanogen" bands with first heads at λλ 4216, 3883, and 3590. The band at λ 3883 is by far the strongest; the point most sensitive to changes in luminosity seems to be the region around and between the two heads at λ 3871 and λ 3883. Of the two other bands, that at λ 3590 seems to be somewhat stronger. The band at λ 4606 seems to be too faint to be identified with certainty.

The spectra of the following stars were photographed; the spectral types and absolute magnitudes are from the new list of spectroscopic parallaxes[3] by Adams and his collaborators.

[1] *Astrophysical Journal*, **49**, 289, 1919; *Upsala Universitets Årsskrift*, 1920.

[2] This instrument is described by W. H. Wright in the *Lick Observatory Bulletin*, **9**, 52, 1917.

[3] *Mt. Wilson Contr.*, No. 199; *Astrophysical Journal*, **53**, 13, 1921.

χ Leonis	F5,	0.8	o_2 Eridani	K1,	6.2
39 Leonis	F8,	3.7	Cin. 1712	K3,	6.4
Lal 19896	G1,	4.5	α Tauri	K4,	1.0
Boss 2338	G3,	−1.0	β Cancri	K4,	0.4
Lal 21008	G6,	5.2	Lal 13284	K5,	7.1
ϵ Virginis	G6,	0.0	Boss 2935	Ma,	10.5
ϵ Geminorum	G8,	−1.4	Boss 1868	Ma,	−0.2
β Geminorum	G9,	1.7	α Orionis	Ma,	−3.8
θ_1 Tauri	G9,	1.4			

The development of the bands is at a maximum for giant stars of types between G5 and K5, and diminishes strongly toward the F and M types. In the dwarf stars of all types the bands are only faintly developed.

The absorption in the band at λ 3883 has been investigated here for stars of known spectroscopic parallax by means of the 6° objective prism on the 10-inch refractor. The method employed is analogous to that used for the A stars described in the first part of this paper. This investigation was carried out before the identification with the "cyanogen" band had been made. The two regions of the spectrum immediately adjacent to λ 3889 were compared in series of images taken with decreasing exposure-time to determine the exposure-ratio for equal photographic density in the two regions. The maximum effect seems to be in spectral types G8–K0, where the difference in exposure-ratio between average giants and dwarfs corresponds to nearly a magnitude. The effect decreases rapidly from G5 to G0. The Cepheids and pseudo-Cepheids have a somewhat peculiar position with even less contrast at λ3889 than dwarfs of the same spectral type. The smoothness of the spectrum around the Hζ line for these stars is probably due to the disappearance of the wings of the strong absorption lines following the advance of the spectrum toward Miss Maury's *c*-stage.

There is a very decided disadvantage, however, in the fact that the region in question is situated so far in the violet that for the redder stars the exposures must be made considerably longer. The band at λ 4216, though weaker, is much more favorable in this particular, and an investigation of it was therefore begun with the same instrumental arrangement. The region most strongly affected by this band lies between $\lambda\lambda$ 4144 and 4184.

This region of 40 A width, corresponding to about 0.09 mm on the plate, has been compared with that at λ 4227–λ 4272 as standard. The regions in question are easily identified with sufficient accuracy by estimating the distances from the G-band and the line λ 4227, which are conspicuous features of the spectra even with very low dispersion. For the dwarfs of types K5–M the intensity in the second region mentioned is probably somewhat modified by the wings of λ 4227 which produce an additional effect of luminosity on the exposure-ratio in the same direction as the effect of the "cyanogen" absorption in the first region. No special efforts were made to determine Schwarzschild's constant for the plates. The results obtained from the investigation of the A stars indicate that generally the changes in the constant are small and do not increase materially the probable error of the measured exposure-ratio, but that rather large changes may occur, especially from one emulsion to another. In this case several stars were measured on each plate, and systematic errors of large size ought to show in the residuals from the curves. The means of the residuals for the individual plates are found to be small, however.

As before, the plates used were Seed 23, size 8×10 inches, with X-ray developer. Images more than 3.6 inches from the center were excluded. The focus was set near λ 4200, and is tolerably constant over most of this area. Even within the adopted limit of distance, however, some images must be excluded by reason of distortion or bad focus, because the field has not exact symmetry of rotation with reference to the plate center.

A scheme for classifying the objective-prism spectra was developed by using types determined by Adams and his collaborators as standards. The classification is based on the appearance and relative strength of the hydrogen lines, the K line of calcium, the λ 4227 line of calcium, the titanium-oxide bands, and the general energy-distribution in the spectrum. Since the decrease of the hydrogen lines and the increase of λ 4227 with advancing spectral type proceed more rapidly in the dwarf series, while the increase of redness takes place faster in the giant series, and since λ 4227 has enormous strength in dwarfs of types K5–M, these characteristics

TABLE VII

Boss or Cin. No.	Harv. Vis. Mag.	Spectrum	*M*	log *E*	*n*	Spectrum Obj. Prism
3274.....	6.5	G8	0.4	0.25	6	G8*g*
3298.....	4.8	K3	0.6	0.27	8	K0*g*
3326.....	5.9	K0	4.2	0.16	4	K2
3344.....	6.5	G8	0.9	0.26	7	G8*g*
3362.....	4.9	Ma	−0.3	0.18	8	Mag
C 1657.....	7.7	K2	6.4	0.14	4	K0*d*
3408.....	6.2	K4	0.5	0.25	4	K5*g*
3537.....	5.7	K3	0.4	0.25	4	K2*g*
3554.....	6.3	F9	4.3	0.12	6	G5*d*
4131.....	5.5	K2	1.1	0.27	5	K2*g*
4137.....	5.6	G2	4.7	0.12	7	G5*d*
4147.....	3.3	G6	0.0	0.19	5	G5*g*
4257.....	6.7	G8	4.7	0.13	7	K0*d*
4264.....	5.4	K5	0.8	0.24	7	K5*g*
4315.....	3.4	K2	0.6	0.26	5	K0*g*
4318.....	5.9	Ma	0.2	0.22	4	Mag
4323.....	5.0	K5	0.2	0.29	5	K2*g*
4327.....	4.4	G2	−1.4	0.27	4	G5*g*
4342.....	7.9	K5	7.9	0.05	6	K5*d*
4370.....	5.3 5.3	K1 K2	6.3 6.3	0.12	7	K0*d*
4372.....	6.7	K4	7.3	0.08	5	K5*d*
4414.....	6.3	K1	1.3	0.24	3	K2*g*
4420.....	4.3	A9	1.6	0.12	4	F2
C 2322.....	7.9	K5	8.0	−0.02	4	K5*d*
Lal 31905.....	7.2	K2	2.1	0.25	4	K5*g*
4571.....	4.3 5.0	K0 K4	5.8 7.8	0.10	6	G8*d*
4580.....	4.7	G6	−0.1	0.27	6	G8*g*
4595.....	5.7	F4	2.8	0.12	6	F5
4618.....	6.1	G2	−0.3	0.20	6	G5*g*
4635.....	4.9	G5	−0.5	0.20	9	G0*g*
4638.....	3.4	G8	2.5	0.15	5	G8
A. G. Leip. 8449.....	7.6	K2	0.8	0.33	5	K2*g*
4660.....	5.7	G0	−0.8	0.22	9	G0*g*
C 2420.....	7.9	G5	5.5	0.11	5	G5*d*
C 2421.....	8.5	G9	5.2	0.10	6	G5*d*
A. G. Leip. 8507.....	8.9	G2	5.7	0.14	5	G5*d*
4678.....	5.4	G9	1.7	0.21	5	K0*g*
A. G. Leip. 8585.....	7.6	Ma	1.5	0.16	3	Mag
4688.....	6.5	G8	2.8	0.22	3	K0*g*
4713.....	5.4	F0	1.6	0.11	6	A–F

TABLE VII—*Continued*

Boss or Cin. No.	Harv. Vis. Mag.	Spectrum	M	log E	n	Spectrum Obj. Prism
C 2456.....	8.8	Mb	10.7	−0.09	3	Mb*d*
4800.....	4.5	Mb	−1.2	0.15	8	Mb*g*
4817.....	5.4	F8	0.3	0.13	5	F8
4823.....	4.2	K1	1.0	0.28	11	K0*g*
C 2495.....	6.0	G4	3.7	0.10	10	G5*d*
4892.....	6.8	G4	5.5	0.13	3	G2*d*
4893.....	6.6	G6	5.7	0.12	3	G2*d*
4898.....	5.7	G8	−1.4	0.28	12	G8*g*
4910.....	6.3	G8	1.5	0.24	2	G5*g*
4912.....	4.5	K0	0.2	0.29	8	K2*g*
4920.....	6.0	F4	2.8	0.12	3	F8
4940.....	4.6	K4	0.5	0.26	4	K3*g*
4950.....	5.2	G8	5.1	0.13	5	G5*d*
5009.....	4.8	G9	5.8	0.09	6	G8*d*
5027.....	4.4	G8	0.0	0.22	6	G8*g*
5052.....	3.8	Ma	0.2	0.11	5	Ma*g*
5079.....	4.0	G2	0.7	0.13	6	G5*g*
5089.....	4.9	G9	0.8	0.18	5	G8*g*
5093.....	3.9	G8	3.9	0.13	5	G8
5098.....	5.8	G1*p*	−2.5	0.20	4	G5*pg*
5118.....	3.7	K6	0.5	0.25	6	K5*g*
5125.....	5.6	Mc	0.5	0.14	5	Mb*g*
5129.....	4.6	Mb	0.3	0.13	8	Mb*g*
C 2616.....	7.2	G8	5.6	0.13	2	G8*d*
5143.....	5.6	G8	0.0	0.27	2	K0*g*
5180.....	5.7	K2	5.9	0.13	6	K2*d*
5187.....	4.0	G9*p*	−2.0	0.23	4	G*pg*
5219.....	6.1	G2	5.2	0.06	3	G5*d*
5229.....	2.3	G0*p*	−3.0	0.24	5	G*pg*
C 2672.....	6.3	G8	6.2	0.12	4	G8*d*
5321.....	6.0	F5	3.6	0.13	2	F5
5346.....	3.6	G8	3.2	0.18	5	G8*g*
C 2707.....	8.6	Ma	9.4	−0.01	3	M*d*
C 2718.....	6.6	F7	3.9	0.10	9	F5
5420.....	6.2	G8	1.5	0.26	7	K0*g*
5431.....	3.9	K2*p*	−2.9	0.24	4	K*pg*
5433.....	5.6	K7	8.3	0.09	9	K5*d*
5434.....	6.3	K8	8.7	0.05	7	Ma*d*

alone will suffice in a great number of cases to determine the spectral type and to decide whether the star is a giant or a dwarf. From type G0 on, however, the degree of the "cyanogen" absorption must be taken into account, and the approximate luminosity will

be obtained with the aid of log *E*. The spectra used are narrow and, in what concerns the visibility of the lines, are inferior to widened images, but this has been at least partly compensated by the large number of images with different intensities usually available. For spectral types Go–Ko both the hydrogen lines and λ 4227 are only faintly visible, but a fairly good discrimination between the stages Go, G5, and G8–Ko seems to be possible. Four Cepheids and pseudo-Cepheids among the stars observed were recognized as peculiar stars mainly by the abnormal strength of the hydrogen lines.

The results of the measurements and classifications are given in Table VII. The first four columns contain data from the list of spectroscopic parallaxes. The fifth column gives log *E*, the logarithmic exposure-ratio for equal intensities in the two regions λ 4144–λ 4184 and λ 4227–λ 4272, the former being situated in the densest part of the "cyanogen" band; *n* is the number of images used for determining log *E*. The last column gives the spectral type as determined from the objective-prism spectra, in most cases without knowledge of the more accurate values of the type in the third column. An added *g* or *d* indicates a giant or a dwarf, respectively. This estimation of luminosity is made mainly from the value of log *E* according to the curves in Figures 3 and 4, the value of the spectral type estimated from the objective-prism spectra being used as argument. The individual results given in the last column are therefore directly dependent only on the data secured from the low dispersion spectra. The good agreement between these results and the spectral type and absolute magnitude in the third and fourth columns seems to give promise of the usefulness of the method.

Figure 3 gives the relation between log *E* and the absolute magnitude for stars of types A9 to K2, Figure 4 for types K2–Mc. The curves drawn representing the different types have only provisional significance. Figure 5 gives log *E* as a function of the spectral type, different degrees of luminosity being represented by different notations. It is evident from these diagrams that the effect of the "cyanogen" band reaches a maximum near Type K2 and decreases toward Go and M. The analogy between

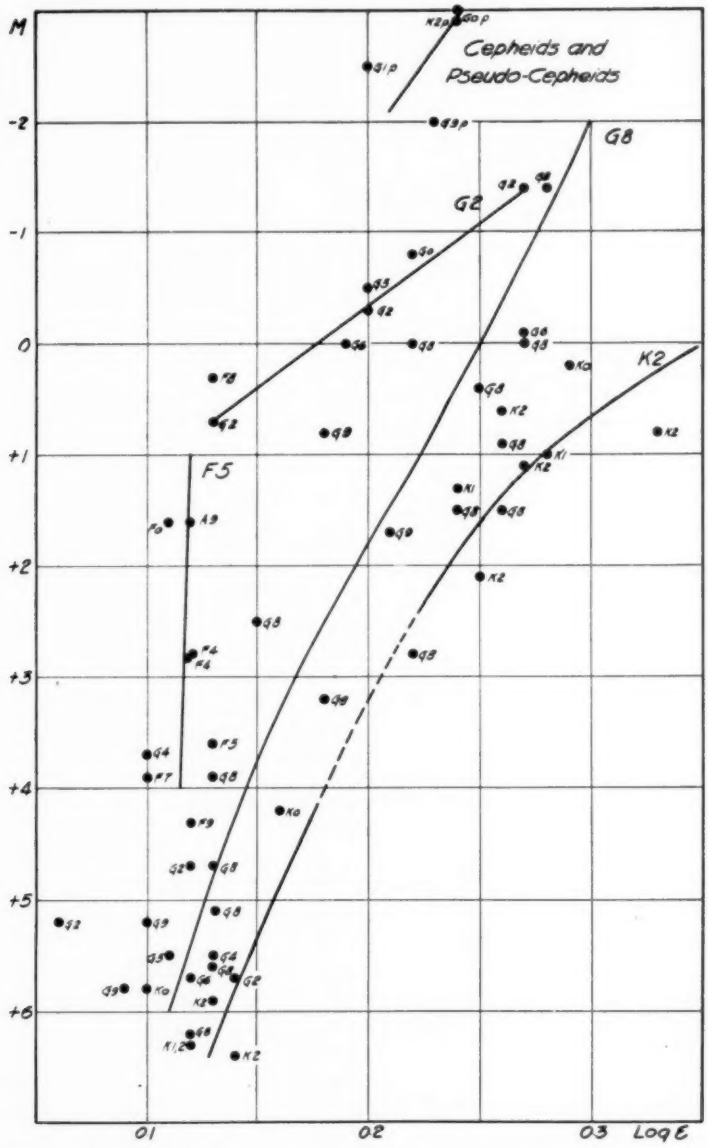

FIG. 3.—Logarithmic exposure-ratio for the regions λλ 4144–4184, 4227–4272, and absolute magnitude. Spectral types A9–K2.

Figure 5 and the diagram for the minimum wave-length of the grating-spectra[1] is very striking. The advantage of the present diagram, however, lies in the circumstance that the effect of the

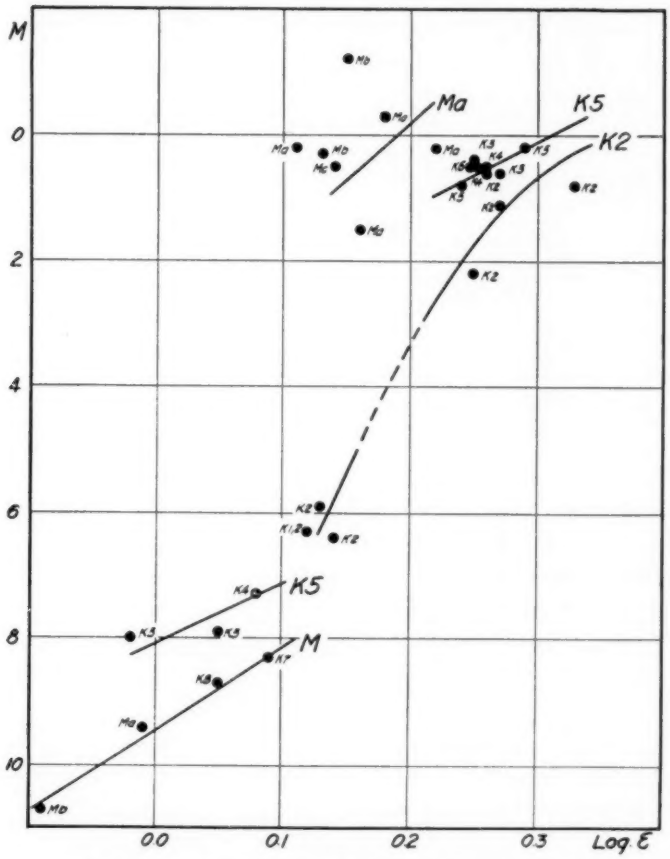

FIG. 4.—Logarithmic exposure-ratio for the regions λλ 4144–4184, 4227–4272, and absolute magnitude. Spectral types K2–Mc.

regular change in the energy-distribution in the spectrum due to the change of temperature in the spectral series has practically no effect on log E, owing to the closeness of the two compared regions. Except for the position of the Cepheids and pseudo-Cepheids the

[1] *Loc. cit.*

correspondence is nearly perfect between Figure 5 and a similarly arranged diagram for the absorption in the λ 3883 band. The variation in log E for the stronger ultra-violet band is, however, about 2.3 times as large as for the λ 4216 band. The increase of the effect from G8 to K2 probably explains the correlation of the absorption in the bands with the line λ 4227 found by Miss Maury in type XV.

The rapid decrease of the effect toward Go, especially for stars with M between 0 and +1, is a disadvantage for a more accurate determination of luminosity. On the other hand, stars of this brightness, between types Fo and G2, seem to be very scarce. Most of the giants in this interval of spectral type given in the list of 1646 spectroscopic parallaxes, for example, are Cepheids or pseudo-Cepheids and of much higher luminosity.

The Cepheid variables and the pseudo-Cepheids seem to have an abnormally small absorption in the regions of the "cyanogen" bands; it is somewhat stronger, however, in the region of the λ 4216 band than in that of the λ 3883 band. In the case of the former band the stars still fall among the ordinary giants in the diagram in Figure 5, but in the case of the ultra-violet band, as mentioned before, they lie even below the dwarfs.

From K2 to M there is a general decrease of log E; the difference between giants and dwarfs is still well accentuated. The extent to which this difference is still due to "cyanogen" absorption in type M seems to be uncertain. For this type the line λ 4227, strong in the dwarfs, and the hydrogen lines, which are strong in the giants,[1] give a reliable criterion of the order of magnitude of the luminosity. The titanium-oxide bands characterizing type M are generally well developed in the objective-prism spectra. An observation of importance to be made from Figure 5 is that stars with log $E > 0.17$ are giants.

An application of the preceding principles to stars of faint apparent magnitudes in a few rich regions has been made with a small focal-plane spectrograph attached to the 60-inch reflector. The spectrograph has a 64° U.V. prism; the focal lengths of collimator and camera are about 6 inches. The dispersion gives 1.51

[1] See various papers by Adams and his associates on the spectroscopic method.

mm between Hγ and Hδ, and is 2.8 times that of the objective-prism spectra. The slit was removed, thus giving a field about $9' \times 9'$, of which about two-thirds is in good enough focus to be used. The objects investigated were the clusters M 11 and 13, and a few fields in Selected Areas Nos. 63 and 64 in the Cygnus region. A list of the photographs is given in Table VIII. Seed 30 plates were used, except for No. 33, which is Seed 23. All the plates were taken during good seeing, estimated about 5 and 6 on a

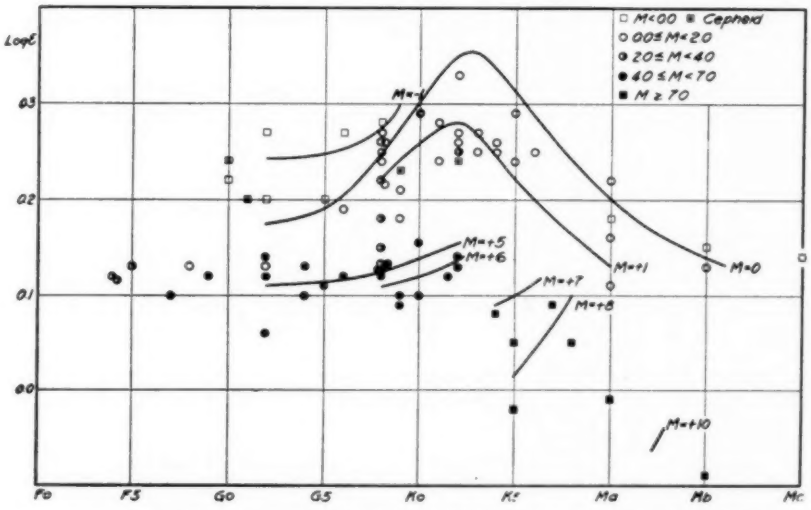

FIG. 5.—Logarithmic exposure-ratio for the regions λλ 4144–4184, 4227–4272, and spectral type.

scale of 10 in most cases. The last column gives for the Selected Area plates the number of the star in the Harvard Durchmusterung[1] which is nearest to the center of the field.

Messier 11, R. A. 18^h46^m, Decl. $-6°23'$ (1900).—This well-known open cluster, situated in the bright galactic clouds in Scutum Sobieski, is very rich in stars, several hundred being brighter than the fourteenth magnitude. Magnitudes and colors for a large number of stars in the cluster have been determined by Shapley.[2]

[1] "Durchmusterung of Selected Areas," *Harvard Annals*, **101**, 1918.

[2] *Mt. Wilson Contr.*, No. 126; *Astrophysical Journal*, **45**, 164, 1917.

Table IX gives the results of the classification of spectra for eighty-five stars in the cluster. A great many stars which are sufficiently bright have their images superposed, thus making estimates of their spectral types impossible. For early types the classification is based on the strength of the K line relative to the hydrogen lines; for the late types the spectra have been compared with spectrograms of stars with known spectroscopic parallaxes, photographed with the same instrumental arrangement on the same

TABLE VIII

LIST OF PLATES

Plate No.	Object	Date	Exposure-Time	Harvard No.
33	Sel. Area 63	1921 May 10	$1^h 43^m$	527
35	Sel. Area 63	May 11	2 55	527
46	Messier 11	Aug. 2	30	.....
47	Messier 11	Aug. 2	2 0	.....
48	Messier 11	Aug. 2	5 1	.....
49	Sel. Area 64	Aug. 2	4 0	666
50	Messier 13	Aug. 3	1 30	.....
51	Sel. Area 64	Aug. 3	4 2	509
65	Sel. Area 64	Sept. 4	3 30	1029
75	Sel. Area 64	Sept. 5	4 37	454

kind of plates. The numbers are those of Stratonoff[1]; the photographic magnitudes are according to Shapley. The letter *d* after the spectral type means for an A-type star that the spectrum shows a marked energy-fall at λ 3907, thus indicating a relatively low luminosity ($M > 1.5$); for later types a *g* indicates giant characteristics, in this case strong "cyanogen" absorption in the region λ 4144–λ 4184.

The giants of types Ko–K2 in Table IX have nearly the same apparent magnitude. The estimate $M = 0$ for their absolute magnitude seems to be fair, judging from the very marked "cyanogen" absorption in their spectra. The mean photographic magnitude of the four stars Nos. 252, 423, 666, 674, is 13.32. With a mean color-index somewhat exceeding unity, we find the modulus $M - m$ to be nearly equal to -12.

[1] *Publications de l'Observatoire de Tachkent*, No. 1, 1899.

The two brightest A stars in Table IX, Nos. 545 and 437, show sharp lines suggesting very high luminosity. With the value $M-m=-12$ their absolute magnitudes would be -2.8 and -0.5,

TABLE IX

MESSIER 11

No.	Pg. Mag.	Spectral Type	No.	Pg. Mag.	Spectral Type	No.	Pg. Mag.	Spectral Type
193	.....	A0	365	13.32	A0	571	13.16	A0
197	.....	A2	366	13.02	A0	579	(Triple)	A0*d*
204	13.38	A0	375	12.54	A0	581	11.85	A0
214	13.48	A0	377	13.64	A0	583	13.84	A2*d*
221	13.07	A0	381	12.44	A0	590	13.04	(G8*g*)?
224	13.56	A0*d*	405	13.18	A0	592	14.22	A0*d*
227	12.06	A0	407	11.93	A0	597	12.78	A2
232	12.22	A0	423	13.46	K2*g*	598	13.02	G8*g*
236	13.07	A0	425	12.62	A2	599	12.40	A0
237	11.74	A0	426	13.21	A0*d*?	601	11.69	F?
245	12.15	A0	437	11.46	A0	616	13.89	A2*d*
252	13.08	K*g*	439	13.64	A0	617	13.25	A0
267	13.26	A0	460	12.97	A0	630	13.54	A0
273	13.00	A0	462	12.79	A0	631	12.46	A0
274	14.32	A2*d*?	464	13.94	A0	633	13.37	A0
280	13.01	G8*g*	468	13.91	A2*d*?	635	13.23	A0
285	13.08	A0	486	12.85	A0	647	12.66	A0
288	12.75	A3	487	12.39	A2	662	12.47	A0
297	12.44	A0	490	13.12	A0	663	11.90	A5*d*
307	14.20	A2*d*	491	12.33	A0	666	13.45	K0*g*
310	13.04	A0*d*?	494	12.32	A0	672	12.50	A0
322	13.46:	A0	511	.....	G8*g*	674	13.28	K2*g*
325	13.04	A2	519	13.30	A0	704	.....	A0*d*?
331	12.24	A0	525	12.08	A0	715	.....	A0
336	12.08	A0	529	13.48	A0*d*?	762	.....	K2*g*
341	12.42	A0	545	9.3:	A5	.....	.....	.....
345	12.06	A0	548	12.23	A0	.....	.....	.....
356	12.60	A0	552	14.20	A0*d*?	.....	.....	.....
362	13.68	A0	568	13.31	A0	.....	.....	.....
364	12.73	G8*g*	569	13.26	A0	.....	.....	.....

respectively. The remaining A stars have fairly broad hydrogen lines. Between photographic magnitude 11.8 and 13.5 the energy-distribution around λ 3907 was found to be smooth, except for star No. 663; with the value of $M-m$ found above, those limits will correspond to $M=-0.2$ and $M=+1.5$. The faintest A stars in the tables show indications of a more abrupt fall of intensity at

λ 3907. For Nos. 224, 307, 592, 616, where this is most noticeable, we find for M the values of 1.6, 2.3, 2.2, 2.1, respectively, which are in good agreement with the results for the A stars in the first part of this paper. The A5 star No. 663, photographic magnitude 11.90, is probably a relatively near star, superposed on the cluster. The spectra of the four faint A stars just mentioned are rather weak, and no definite conclusions can be drawn from those stars alone, but the indications are that the value $M-m=-12$ deserves a fair degree of confidence. Adopting this value we find for the parallax of the cluster $\pi=0''.00025$, corresponding to a distance of 4000 parsec or 13,000 light-years. Shapley,[1] obtaining a value by an entirely different method, finds $\pi=0''.00014$. In view of the uncertainties affecting the two determinations the difference cannot be considered as by any means excessive.

Globular clusters.—On Plate No. 50 three of the brightest stars in the globular cluster Messier 13 are sufficiently well separated from other stars to permit an examination of their spectra. They are the Scheiner Nos. 47, 63, 127. Shapley[2] gives the photographic magnitudes 13.45, 13.70, 13.58, and the color-indices 1.41, 1.16, 1.42. With the parallax $=0''.000090$[3] the absolute magnitudes are -3.19, -2.69, -3.07, respectively. These stars seem to be of a spectral type not far from Ko, but show a much smaller absorption in the region of the "cyanogen" band than the late-type stars in Messier 11. This agrees with the degree of luminosity found by Shapley, if the stars are pseudo-Cepheids; if they are ordinary giants we should estimate the absolute magnitude around $+1$. Low dispersion slit spectrograms of some stars in Messier 13 on a plate taken by Mr. Pease, and of stars in Messier 3 on a plate taken by Mr. Sanford, exposure-times 30^h and 20^h, respectively, seem rather to confirm the high degree of luminosity. The stars of earlier type on the first mentioned plate seem to have no appreciable change of intensity at λ 3907, and for the late-type stars on the latter plate, especially for the star von Zeipel No. 752, photographic magnitude 13.96, color-index 1.42, absolute magnitude according to

[1] *Proceedings of the National Academy of Sciences*, **5**, 344, 1919.

[2] *Mt. Wilson Contr.*, No. 116.

[3] Shapley, *Mt. Wilson Contr.*, No. 152; *Astrophysical Journal*, **48**, 154, 1918.

Shapley − 3.18, the depression between λ 4100 and λ 4227 seems to be centered around Hδ and λ 4172 rather than in the "cyanogen" band, observations in favor of a classification in the Pseudo-Cepheid type.

Selected Areas

No. 63, R.A. 19^h0^m, Decl.+30°0′, (1900)
No. 64, R.A. 19^h58^m, Decl.+30°0′.

TABLE X

SELECTED AREAS 63 AND 64

Harv. No.	Pg. Mag.	Spectral Type	Harv. No.	Pg. Mag.	Spectral Type	Harv. No.	Pg. Mag.	Spectral Type
Area 63			504....	14.5	Ao	710...	12.87	Ao
511....	13.20	K3*g*	524....	14.0	A2	719...	14.40	K2*g*
513....	12.61	G5*d*	535....	12.5	Ao	726...	14.26	K2*g*
518....	9.61	Ao	581....	12.5	B9	733...	11.70	Ao
519....	10.50	A5	583....	12.0	Ao	744...	13.06	F8
535....	12.43	G0*d*						
			603....	13.6	A2	748...	13.31	Ao
539....	11.86	A5*d*	611....	13.8	F8	767...	12.10	G5*d*
546....	13.43	K2*g*	619....	14.6	Ao	1005...	11.3	Ao
			623....	14.2	Ao	1007...	10.6	Ao
Area 64			629....	13.7	Ao	1017...	13.1	K0*g*
430....	14.6	A						
435....	13.2	K2*g*	646....	11.3	G8*g*	1020...	13.9	K2*g*
438....	13.8	F5*g*	661....	14.1	Ao	1021...	13.6	B?
442....	13.2	G8*d*	667....	14.18	K5*g*	1026...	10.9	Fo
443....	14.0	F8*d*	668....	13.0	F8*d*	1027...	13.8	G0*d*
			671....	14.22	K0*g*	1028...	13.3	Ao
445....	14.0	K5*g*						
448....	13.9	G5*d*	677....	13.31	Ao	1104...	11.8	A2*d*
476....	13.5	G5*d*	680....	13.86	K0*g*			
479....	14.1	Ao	682....	13.64	G5*d*			
486....	12.7	G5*d*	702....	13.81	K2*g*			
			706....	10.52	A2			

The results of classification for fifty-two stars in Selected Areas 63 and 64 are given in Table X. The numbers are according to the Harvard Durchmusterung. The photographic magnitudes for the central fields are according to unpublished Mount Wilson measures; for stars more distant from the center of the area the Harvard magnitudes have been reduced to the Mount Wilson system, and are given to only one decimal.

These two areas are situated in very rich galactic regions. There are eleven late-type stars classified as dwarfs in Table X,

which as a group may be considered to be the nearest stars in the region. Of the remaining more distant stars twenty-eight are fainter than the thirteenth magnitude, twelve of which are found to be giants of types K0–K5, a remarkably high percentage. These late-type giants range in photographic magnitudes from 13.1 to 14.4, with a mean of 13.8. With a mean color-index of 1.2 and an absolute visual magnitude of 0.5 they should be at the same distance as M 11, about 4000 parsecs. If M 11 is associated with the galactic clouds in Scutum, the inference would be that these giant stars together with some of the A stars are members of the distant galactic cloud in Cygnus.

I wish to express my deep feeling of obligation to Director Hale for the opportunity to undertake this investigation during a stay of more than a year at the Observatory, and also to several members of the staff, especially Mr. Adams and Mr. Seares, for much valuable information and advice. I also wish to express my gratitude to Director Campbell for a month's stay at the Lick Observatory, and to Mr. Wright for his kind help and instruction in the work with the Crossley reflector.

MOUNT WILSON OBSERVATORY
October 1921

THE THEORY OF IONIZATION AND THE SUN-SPOT SPECTRUM[1]

By HENRY NORRIS RUSSELL[2]

ABSTRACT

Saha's theory of temperature ionization, assuming the ionization to be due to temperature alone, leads to the relation: $Px^2/(1-x^2)=K$; that is, the ratio of the fraction of atoms ionized (x) to the fraction un-ionized ($1-x$), times the partial pressure of the free electrons $[Px/(1+x)]$, is equal to K, a function only of the ionization voltage I and of the absolute temperature T, namely: $\log_{10} K=-5036\, I/T+2.5 \log_{10} T-6.5$. The theory is here *extended to mixtures of elements* by putting the partial pressure of the electrons equal to $P\bar{x}/(1+\bar{x})$ where $\bar{x}$ is the fraction of ionization for all the atoms present, and P, as before, is the total pressure. Hence, $Px_1\,\bar{x}/(1-x_1)(1+\bar{x})=K_1$; $Px_2\,\bar{x}/(1-x_2)(1+\bar{x})=K_2$; etc.; and the ratio of $x_1/(1-x_1)$ to $x_2/(1-x_2)$ is merely K_1/K_2. Therefore the degree of ionization is always higher for an element of easy than for one of difficult ionization, and the relation between the two depends only on (I_2-I_1) and on T, and not on the pressure or on the relative concentrations. Saha's *formulae for second-stage ionization* are corrected. Only two successive stages of ionization can be present in any considerable proportions at the same time. Strong *evidence in favor of the theory* is afforded by a comparison of the sun-spot and the solar spectrum, since the predictions of the theory with reference to the relative intensity in the hotter and the cooler spectrum, of lines associated with ionized and un-ionized atoms, are found to be in general agreement with the facts. However, discrepancies such as the presence of too large a fraction of ionized Ba atoms, suggest the need of some modification of the theory; and it is evidently incomplete, since it neglects the effects of radiation on ionization.

Relative intensity in solar and sun-spot spectra of series lines.—The lines of the *alkali metals* (all of which are due to the neutral atoms) are *greatly strengthened* in the spot spectrum. Na ($I=5.11$ volts) exhibits its principal and subordinate series, K (4.32) the principal series alone, and Li (5.37) and Rb (4.16) the leading pair of the principal series, and only in the spot spectrum. The corresponding lines of Cs are out of reach in the infra-red.

Among the *alkaline earths* the lines of the *neutral atom* for Ca (6.08 volts) are strengthened in the spots to a degree which is greatest for the principal series and least for the combination series. The corresponding lines of Sr (5.67) are faint in the sun and much strengthened in spots. For Ba (5.12 volts) they are absent both in sun and spots. The *enhanced lines* arising from the ionized atoms (Ca+, Sr+, Ba+) are strong in both spectra.

For Mg (7.65 volts) the lines of the *neutral atom* are little affected, except that the fundamental line $1S-2p_2$ is stronger, while for Zn (9.4 volts) the arc lines are much weaker in the spot spectrum.

These results are all in agreement with Saha's theory. The lines of Li and Ba, however, are much weaker than the theory indicates.

Relative intensity of arc and enhanced lines.—It is pointed out that enhanced lines of easily ionized elements can be produced with less excitation than arc lines of difficultly ionized elements. This fact explains many apparent anomalies in astrophysical spectra.

Ionization potential of the elements as a function of atomic number.—Since from the relative behavior in sun, spot, and furnace spectra, of the lines of *Ca, Sc, Ti, V, Cr,*

[1] *Contributions from the Mount Wilson Observatory*, No. 225.

[2] Research Associate of the Mount Wilson Observatory.

Mn, Fe, Co, Ni, Cu, and Zn, the elements as arranged in the order of their atomic numbers, are progressively more difficult to excite spectroscopically, the ionization potential probably increases regularly from 6 volts for Ca to 9.4 volts for Zn, and this result combined with other data suggests that, *in general*, the ionization potential is a periodic function of the atomic number.

Identification of certain solar lines.—Lines of Rb have been detected in the sun for the first time. Also several faint Na lines have been identified and Datta's identifications of Na and K lines have been partly confirmed, partly not.

Tables of the series lines of Li, Na, K, Rb, Cs, Ca, Sr, Ba, Mg, and Zn are given with intensities in sun and spot, and wave-lengths in international units to 0.01 A.

Dr. Megh Nad Saha, in an important series of papers,[1] has shown the great importance of the modern thermodynamic theory of ionization in astrophysical and spectroscopic researches. In particular, he makes certain predictions about the behavior of the lines of the alkali metals in the spectra of sun-spots. The present communication deals with certain points connected with the theory, especially when many kinds of atoms are present at the same time, and with the observational verification of the predictions, which has been complete.

I. THE THEORY OF IONIZATION WHEN ATOMS OF SEVERAL KINDS ARE PRESENT

The equation of the "reaction isobar" as given by Saha[2] in the case of the ionization of a gas consisting of atoms of but a single kind is

$$\log \frac{x^2}{1-x^2}P = \log K = -\frac{U}{4.571\,T} + 2.5 \log T - 6.5 \qquad (1)$$

where the logarithms are to the base 10, P is the pressure of the gas, x the fraction of all the atoms which are ionized, T the absolute temperature centigrade, and U the "heat of ionization" in calories per gram-molecule. For the gas in question, the latter is proportional to the ionization potential, I, being 23,020 calories if I is one volt, so that we may also write

$$\frac{x^2}{1-x^2} = \frac{K}{P}, \qquad \log K = -\frac{5036\,I}{T} + 2.5 \log T - 6.5. \qquad (2)$$

[1] "Ionization in the Solar Atmosphere," *Philosophical Magazine*, **40**, 472, 1920; "Elements in the Sun," *ibid.*, **40**, 809, 1920; "On the Temperature Radiation of Gases," *ibid.*, **41**, 267, 1921; "On a Physical Theory of Stellar Spectra," *Proceedings of the Royal Society*, A, **99**, 135, 1921.

[2] *Philosophical Magazine*, **40**, 479, 1920.

If atoms of several different kinds, all capable of ionization, are present, the situation is somewhat more complicated. To use equation (1), introducing for P the value of the partial pressure of the vapor of each element separately, is inadmissible, since one of the products of the reaction—free electrons—is produced by all the ionizations. But if p' is the partial pressure of the ionized atoms of any sort, p that of the corresponding non-ionized atoms, and p'' that of the free electrons, we will always have

$$\frac{p'p''}{p}=K \tag{3}$$

when K is given by (2).

If now atoms of several kinds are present in the relative numbers $a_1, a_2, a_3 \ldots$ and the fractions $x_1, x_2, x_3 \ldots$ of these are ionized, the whole number of atoms and free electrons present will be $a_1(1+x_1)+a_2(1+x_2) \ldots$ and the partial pressures for element 1 will be

$$p_1'=\frac{a_1x_1}{\Sigma a+\Sigma ax}P \qquad p_1=\frac{a_1(1-x_1)}{\Sigma a+\Sigma ax}P$$

(P being as before the whole pressure of the gas), while for the free electrons we will have

$$p''=\frac{\Sigma ax}{\Sigma a+\Sigma ax}P=\frac{\bar{x}}{1+\bar{x}}P$$

where

$$\bar{x}=\frac{\Sigma ax}{\Sigma a}. \tag{4}$$

$\bar{x}$ is therefore the fraction of all the atoms present which are ionized, and may be regarded as a weighted mean of the values of x for the individual elements.

Equation (3) then becomes for the various elements

$$\frac{x_1}{1-x_1}\frac{\bar{x}}{1+\bar{x}}=\frac{K_1}{P}, \qquad \frac{x_2}{1-x_2}\frac{\bar{x}}{1+\bar{x}}=\frac{K_2}{P}, \ldots \tag{5}$$

where K_1, K_2 are given by (2) with the ionization potentials for the elements concerned.

We then have

$$\frac{x_1}{1-x_1}=\frac{K_1}{K_2}\frac{x_2}{1-x_2}, \qquad \log\frac{K_1}{K_2}=5036\,\frac{I_2-I_1}{T} \tag{6}$$

or, expressed verbally: *The ratios of the number of ionized atoms to that of non-ionized atoms for any two elements in a gaseous mixture bear a fixed proportion to one another, which depends only on the temperature, and is independent of the pressure, the relative abundance of the two elements, or the presence of other elements.*

These latter conditions affect the amount of ionization, without modifying this proportion. The element of lower ionization potential is always the more highly ionized.

We may write (5) in the form

$$\frac{x_1^2}{1-x_1^2}=\frac{K_1}{P_1},$$

where

$$P_1=P\frac{\bar{x}}{x_1}\frac{1+x_1}{1+\bar{x}}. \tag{7}$$

If $x_1>\bar{x}$, that is, if the element is more easily ionized than the average, P_1 will be less than P, and the percentage of ionization will be greater than it would be if this element alone were present, with the same total pressure. For elements ionized with more difficulty than the average, the reverse will be the case.

The ionization will in all cases be less than if this element were present alone with its actual partial pressure, for p″, in equation (3), will always be greater than the partial pressure p_1' of the electrons arising from the dissociation of this one component of the mixture.

When ionization is just beginning all the x's will be small, and we will have approximately

$$x_2=\frac{K_2}{K_1}x_1, \text{ etc.} \qquad \bar{x}=\frac{x_1\Sigma aK}{K_1\Sigma a}$$

$$P_1=P\frac{\bar{x}}{x_1}=P\frac{a_1}{\Sigma a}\left(1+\frac{K_2a_2}{K_1a_1}+\frac{K_3a_3}{K_1a_1}\ldots.\right).$$

The factor outside the parentheses is the partial pressure of constituent 1. It follows that at the beginning of ionization the constituent of lowest ionization potential behaves very nearly as if it alone were present at its actual partial pressure. For constituents more difficult of ionization the initial ionization is less, and may be very much smaller.

When the ionization of any element is nearly complete, we may set $x_1 = 1$ in (7) and find

$$P_1 = P\frac{2\bar{x}}{1+\bar{x}}.$$

The effective pressure for this element is therefore nearly equal to the total pressure (unless a large proportion of atoms of more difficult ionization is present).

For an element of easy ionization the range of temperature or pressure within which both ionized and uncharged atoms will be present in sensible proportions is therefore extended by the presence of other constituents in the mixture.

For an element of difficult ionization, x_1 will be much less than $\bar{x}$ and P_1 greater than P, when ionization is beginning. The range within which both phases are present will in this case be reduced.

If we set

$$\frac{\bar{x}^2}{1-\bar{x}^2} = \frac{\overline{K}}{P}$$

we find easily

$$\overline{K}\frac{1-\bar{x}}{\bar{x}} = K_1\frac{1-x_1}{x_1} = K_2\frac{1-x_2}{x_2} \cdots = \frac{\Sigma aK(1-x)}{\Sigma ax}$$

$$\overline{K} = \frac{\Sigma aK(1-x)}{\Sigma a(1-x)}$$

so that $\overline{K}$ is the mean of the individual K's, weighted in proportion to the number of non-ionized atoms remaining. At low temperatures, the atoms of easy ionization, which have the largest values of K, will contribute most to this mean; but at high temperatures these will be ionized, and the atoms of difficult ionization will contribute most. The effective mean ionization potential, corresponding to the value of $\overline{K}$, will therefore increase with the temperature.

Successive ionization.—Consider now atoms of the same kind, which are susceptible of the loss of two successive electrons. Let the ionization potential for the removal of the first be I and the additional potential for the second be I'. If x is the proportion

of singly-ionized and y of doubly-ionized atoms, that of free electrons will be $x+2y$, and we will have[1]

$$\left.\begin{aligned}\frac{x(x+2y)}{(1-x-y)(1+x+2y)}&=\frac{K}{P}\\ \frac{y(x+2y)}{x(1+x+2y)}&=\frac{K'}{P}\end{aligned}\right\}. \tag{8}$$

The first of these equations may be written

$$\frac{(x+y)^2}{1-(x+y)^2}=\frac{K}{P}\frac{(x^2+2xy+y^2)(1+x+2y)}{(x^2+2xy)(1+x+y)}>\frac{K}{P}$$

also

$$\frac{x^2}{1-x^2}=\frac{K}{P}\left(1-\frac{y(2-x+x^2+2xy)}{(1-x^2)(x+2y)}\right)<\frac{K}{P}$$

while the second may be written

$$\frac{y^2}{1-y^2}=\frac{K'}{P}\frac{x}{1-y}\frac{2y^2+xy+y}{2y^2+xy+2y+x}<\frac{K'}{P}.$$

It follows that the numbers of both neutral and singly-ionized atoms are less than they would be if the second ionization did not occur—the draft upon the singly-ionized atoms being partly made up by fresh ionization of neutral ones; but the number of doubly-ionized atoms is less than it would be if the second ionization was the only one which occurred.

Dividing the second of the equations (8) by the first we find

$$\frac{y(1-x-y)}{x^2}=\frac{K'}{K}. \tag{9}$$

Now

$$\log\frac{K'}{K}=-5036\frac{I'-I}{T}$$

and in all cases which have so far been investigated $I'-I$ is positive, and equal to at least five volts.

[1] These equations differ from Saha's (*Proceedings of the Royal Society*, A, **99**, 143, 1921) which contain x^2 and y^2 in the numerators; but the latter formula would appear to involve the inadvertent assumption that an atom can combine only with an electron previously liberated by another atom which happens to be in the same state of ionization.

Hence K'/K is usually a small quantity, and it is impossible for y and $1-x-y$ to be considerable at the same time; that is, there will not simultaneously be any noticeable proportion of atoms in all three states of ionization. When the temperature is high K'/K increases; but in this case x is small and the number of neutral atoms very small.

At low temperatures we may set $y=0$, when the ordinary equation for single ionization is obtained; at high temperatures we may set $x+y=1$ and have

$$\frac{y^2+y}{2-y^2-y}=\frac{K'}{P}.$$

One or the other of these formulae will always give a good first approximation, provided that a third electron is not lost.

In a mixed gas, containing other constituents, we will have

$$\frac{x\bar{x}}{(1-x-y)(1+\bar{x})}=\frac{K}{P};\quad \frac{y\bar{x}}{x(1+\bar{x})}=\frac{K'}{P};\quad \bar{x}=\frac{\Sigma a(x+2y)}{\Sigma a}. \tag{10}$$

Since the potential for the second ionization is always high, the presence of other elements will in general diminish its amount; but equation (9) will hold in all cases.

Instead of equation (6) we will have

$$\frac{x}{1-x-y}=\frac{K}{K_2}\frac{x_2}{1-x_2}\qquad \frac{y}{x}=\frac{K'}{K_2}\frac{x_2}{1-x_2} \tag{11}$$

where x_2 is the percentage of ionization for any singly ionized element.

2. THE ALKALIES AND ALKALINE EARTHS IN THE SUN-SPOT SPECTRUM

Saha has shown[1] that sodium should be considerably ionized in the reversing layer, potassium almost completely, and rubidium and caesium entirely. As the familiar lines of these elements are absorbed by the neutral atoms only, the faintness of the lines of potassium in the solar spectrum and the absence of those of rubidium and caesium is immediately explained. At the lower temperatures which prevail in the spots he predicts that the lines of

[1] *Philosophical Magazine*, **42**, 812–815, 1920.

potassium should be strengthened, and that those of rubidium, should show faintly.

Observations upon the behavior of sodium in sun-spots have been published by Adams[1] and the presence of lithium has been determined by King.[2] The study of the other alkali metals, which have their lines principally in the red, has been made very easy by the publication of extensive and accurate tables of wave-lengths in the red and infra-red, both in the solar spectrum[3] and in the arc,[4] and by the existence at Mount Wilson of an excellent series of plates of the spot spectrum, extending to λ 8200, taken by Mr. Brackett with the 150-foot tower telescope and 75-foot spectrograph, in the first order, with nicol prism and compound quarter-wave plate. All the available lines of the alkali metals were examined on these plates (or on the photographic map of the spot spectrum for the wave-lengths less than 6600 A) and some additions to the data for sodium and lithium were made.

The results, including those of the earlier observers, are given in Table I. In this table the first column gives the element and series designation of the line (after Sommerfeld). The leading pair of the principal series is denoted by 1s–2p, the next pair by 1s–3p, and so on. The corresponding pairs for the diffuse series are 2p–3d, 2p–4d, and for the sharp series 2p–3s, etc. Lines lying too far in the infra-red to be observed, or too far in the ultra-violet to be observable in spots, are in general omitted, as are also the fainter members of a series when the stronger earlier members are invisible.

The second column gives the wave-lengths found in the laboratory by Datta or by Meggers (or, in one or two cases, by others); and the third, the wave-length of the corresponding solar line derived from Rowland's tables, from Meggers, or from measurements made at Mount Wilson. All wave-lengths are given in

[1] *Mt. Wilson Contr.*, No. 40; *Astrophysical Journal*, **30**, 108, 1909.

[2] *Mt. Wilson Contr.*, No. 122; *Astrophysical Journal*, **44**, 169, 1916.

[3] W. F. Meggers, "Solar and Terrestrial Absorption in the Sun's Spectrum from 6500 A to 9000 A," *Publications of the Allegheny Observatory*, **6**, 12–44, 1919.

[4] W. F. Meggers, "Wave-Length Measurements in Spectra from 5600 A to 9600 A," *Bulletin of the Bureau of Standards*, **14**, 371–397, 1916; S. Datta, "The Vacuum Arc Spectra of Sodium and Potassium," *Proceedings of the Royal Society*, A, **99**, 69–78, 1921.

TABLE I

LINES OF THE ALKALI METALS

ELEMENT	WAVE-LENGTHS		INTENSITY		AUTHORITY	NOTES
	Arc	Sun	Sun	Spot		
Lithium						
1s–2p.....	6707.82	07.83	abs	5	King	1
2p–2s.....	8126.52	.....	abs	abs	Russell	
2p–3d.....	6103.53	.....	abs	abs	King	
Sodium						
1s–2p.....	5895.94	95.94	20	60	Adams	
	5889.97	89.97	30	90	Adams	
2p–3d.....	8194.82	94.84	2	7	Russell	
	8183.30	83.27	3	10	Russell	
2p–4d.....	5688.22	88.22	6	12	Adams	
	5682.67	82.65	5	12	Adams	
2p–5d.....	4982.86	82.82	2	4	Russell	2
	4978.61	78.56	0	1	Russell	3
2p–6d.....	4668.60	68.57	1*N*	3	Russell	4
	4664.86	.....	.....	.....	Russell	5
2p–7d.....	4497.72	97.73	abs	0	Wilson	6
	4494.27	.....	abs	abs	Russell	7
2p–4s.....	6160.72	60.75	3	8	Adams	
	6154.21	54.23	2	9	Adams	
2p–5s.....	5153.85	.....	abs	abs	Russell	8
	5149.27	.....	abs	abs	Russell	9
2p–6s.....	4751.89	51.83	00	2	Adams	
	4748.02	47.98	000	0	Adams	
Potassium						
1s–2p.....	7699.00	99.01	5	13	Russell	
	7664.92	64.92	6	15	Russell	10
1s–3p.....	4047.20	47.19	00*Nd*	0	Russell	11
	4044.14	44.14	0	2	Russell	
2p–4d.....	6965.0	.....	abs	abs	Russell	12
	6936.0	.....	abs	abs	Russell	
2p–5d.....	5832.31	.....	abs	abs	Russell	
	5812.71	.....	abs	abs	Russell	
2p–4s.....	6939.50	.....	abs	abs	Russell	13
	6911.80	.....	abs	abs	Russell	
2p–5s.....	5802.16	.....	abs	abs	Russell	
	5782.77	.....	abs	abs	Russell	

TABLE I—*Continued*

Element	Wave-Lengths In		Intensity In		Authority	Notes
	Arc	Sun	Sun	Spot		
Rubidium						
1s–2p.....	7947.64	47.63	abs	0	Russell	14
	7800.29	00.29	abs	1	Russell	
1s–3p.....	4215.57	.....	abs	abs	Russell	
	4199.85	.....	abs	abs	Russell	
2p–4d.....	7757.80	.....	abs	abs	Russell	
	7619.12	.....	abs	abs	Russell	
Caesium						
1s–2p.....	8943.46	.....	abs	Unknown	.....	15
	8521.12	.....	abs	Unknown	.....	
1s–3p.....	4593.22	.....	abs	abs	Russell	
	4555.31	.....	abs	abs	Russell	
	8079.4	.....	abs	abs	Russell	16
	8016.2	.....	abs	abs	Russell	

NOTES TO TABLE I

1. Much widened in the spot, and showing strong Zeeman effect.

2. Rowland's line 4982.994, for which he gives no origin. Identified by Datta.

3. Rowland 4978.732. Shows, like the last, a conspicuous Zeeman effect. Clearly separated from the iron line 4978.785 which is erroneously identified by Datta as the sodium line.

4. Rowland 4668.749. Shows Zeeman effect. Identified by Datta.

5. Masked by the chromium line 4664.965 (Rowland).

6. A faint line, visible only in the spot, and showing conspicuous Zeeman effect. Measured by Wilson (unpublished) 4497.900 on Rowland's scale. Datta identifies this with Rowland 4497.842, a titanium line which is clearly separated.

7. Datta identifies this with Rowland 4494.356 (00) which is not strengthened in the spot, and is probably not the sodium line. No trace of the latter could be seen. The character of the foregoing lines, and their strong intensification in the spots, support their identification as sodium lines. It is very difficult to get these lines sharp in the laboratory, and Datta's measures, though much superior to earlier ones, demanded long exposures.

8. Datta identifies as Rowland 5153.848 (000). This is a band line showing no Zeeman effect.

9. Datta identifies as 5149.267 (000) Rowland C. This is very little strengthened in the spot and shows no Zeeman effect. No trace of the sodium lines could be found in this vicinity.

10. Practically coincident with a strong atmospheric line in the tail of the A band, but identified by Meggers by means of the sun's rotation. The intensification of this line in the spot spectrum and the Zeeman effect are conspicuous in spite of this blending, which has been allowed for in the estimates of intensity.

11. Identification by Rowland and Datta confirmed.

12. Wave-lengths from Saunders. These lines are abnormally faint in the arc spectrum. Datta identifies as Rowland 6965.320 (000 N) which shows no Zeeman effect and 6926.290 (0000) which is invisible both in sun and spot on the Mount Wilson plates.

13. Identified by Datta as 5832.490 (000) and 5812.942 (0000). These lines are too faint to be visible on the Mount Wilson plates, either in sun or spot.

14 Typical spot lines, diffuse and showing strong Zeeman effect, and greatly resembling the lithium line at λ 6708. Measured by the writer, the first on one plate, the second on two. Probable error of measurement about ±0.01 A. One of the Zeeman components of 7947.63 is clearly separated from the atmospheric line at 7947.766 (Meggers). The other is blended with it.

15. These lines, which are the first pair of the principal series, lie beyond the limits of the region so far photographed at Mount Wilson, and attempts to secure plates showing the spot spectrum in this region have not yet been successful. They are not present in the solar spectrum (Meggers). The absence of the second pair of this series might have been anticipated.

16. This relatively narrow pair is strong in the arc, but does not belong to the known series. There are no strong lines of the subordinate series of caesium in favorable positions for investigation.

International units and Rowland's values have been reduced to this standard with the aid of a correction-curve which was kindly furnished by St. John.

The fourth and fifth columns give the intensity of the lines in sun and spot. The values in the former are taken from Rowland or Meggers; the latter, for the lines previously observed, from Adams or King. For lines newly observed, estimates have been made on the original plates, as nearly as possible upon the same scale as that used by Rowland or Meggers. Cases in which a line is certainly invisible are denoted by "abs." The sixth column gives the authority for the data concerning the spot spectrum (including measures of wave-length when the line does not appear in the spectrum of the photosphere), and the last provides references to the notes which follow.

These results may be summarized as follows:

Sodium is represented in the sun by the principal, diffuse, and sharp series, and all its lines are much strengthened in the spot spectrum. Some of Datta's identifications of faint lines have been confirmed and others modified. The absence of the pair λλ 5153, 5149 in both sun and spot is remarkable, since the following and fainter pair of the series appears to be present.

Potassium is represented by the principal series only. Its lines are greatly strengthened in the spots. Datta's identifications of faint solar lines as belonging to the subordinate series of this element are not confirmed.

Lithium shows in the spot spectrum alone, and only through the leading line of the principal series.

Rubidium, previously undetected in the sun, is definitely proved to be present by the appearance of both members of the leading pair of the principal series in the spot spectrum.

The corresponding lines of caesium lie far in the infra-red, and satisfactory photographs of the spot spectrum have not yet been obtained in this region.

The alkaline earths (for which the ionization potentials are likewise known) are of special interest, because the fundamental members 1s–2p, of the series of enhanced lines, absorbed by the ionized atoms, lie within the accessible portion of the solar spectrum

TABLE II

LINES OF ALKALINE EARTHS IN SUN-SPOTS

ELEMENTS	WAVE-LENGTH	INTENSITY IN		NOTES
		Sun	Spot	
Mg				
1S–2P	2852.13	.....	.....	Out of reach
1S–2P$_2$	4571.11	5	7	1
2P–4D	5528.47	8	8	1
2P–5D	4703.07	10	10	1
2P–6D	4351.94	5	4	1
2P–4S	5711.13	6	6	1
2p–3s	5183.62	30	30	1
	5172.68	20	22	1
	5167.33	15	17	1
2p–4d	3838.28	25	.....	
	3832.31	15	.....	7
	3829.36	10	.....	
Mg+				
1s–2p	2802.70	.....	.....	Out of reach
	2795.52	.....	.....	
Ca				
1S–2P	4226.73	20	25	2
1S–2p$_2$	6572.78	1	10	3
2P–3D	7326.10	2	5	4
2P–4D	5188.85	3	4	4
2P–4S	5512.98	4	5	4
2P–5S	4847.29	0	1	4
3D–4F	4878.13	3	3	4
3D–2P	6717.19	5	8	4
2p–4d	4556.61	2	4	2
	4555.88	3	5	2
	4554.77	5	7	2
	4535.67	4	6	2
	4534.95	5	7	2
	4525.43	4	6	2
2p–2s	6162.18	15	25	2
	6122.22	10	20	2
	6102.72	9	20	2
3d–4f	4585.92	4	6	4
	4585.87	.....	.....	
	4581.41	4	6	4
	4578.57	3	6	4
Ca+				
1s–2p	3968.08	700	700	3
	3933.68	1000	1000	3
2p–3d	8662.42	15	.....	
	8542.48	16	.....	
	8498.32	12	.....	
2p–2s	3736.92	5	.....	7
	3706.03	6	.....	7
Sr				
1S–2P	4607.35	1	3	4
1S–2p$_2$	6892.36	abs	abs	4
2p–3s	7070.7	abs	abs	4
	6878.8	abs	abs	4
	6791.4	abs	abs	4

TABLE II—*Continued*

Elements	Wave-Length	Intensity in		Notes
		Sun	Spot	
Sr				
2p–4d	4971.68	abs	abs	4
	4967.94	.....	.....	5
	4962.28	ooo	oo	4
	4876.06	abs	abs	4
	4872.49	abs	ooo	4
	4852.06	abs	abs	4
Sr+				
1s–2p	4215.50	8	8	2
	4077.73	5	5	2
2p–4d	10915.0	.....	.....	
	10328.3	.....	.....	Out of reach
	10038.7	.....	.....	Out of reach
2p–2s	4305.44	(3)	(5)	5
	4161.79	1	oo	2
Ba				
1S–2P	5535.53	abs	abs	6
1S–2p_2	7911.36	abs	abs	4
Ba+				
1s–2p	4934.07	7	7	4
	4554.04	8	10	4
2p–4d	4166.01	o	o	4
	4130.65	2	2	4
	3891.83	.....	.....	5
2p–2s	4899.97	(2)	(2)	5
	4524.95	o	o	4
Zn				
1S–2P	2139.33	.....	.....	Out of reach
1S–2p_2	3075.90	1	.....	7, 8
2P–3D	6362.35	1	o	9
2p–2s	4810.54	3	o	4
	4722.16	3	o	4
	4680.14	1	oo	4
2p–4d	3346.02	o	.....	7
	3345.58	1	.....	7
	3345.12	2	.....	7
	3303.37	.....	.....	5
	3303.59	1	.....	7
	3282.33	.....	.....	5

NOTES

1. Adams, *Mt. Wilson Contr.*, No. 40, p. 20; *Astrophysical Journal*, **30,** 105, 1909.
2. Adams, *Mt. Wilson Contr.*, No. 22; *Astrophysical Journal*, **27,** 45–55, 1908.
3. Adams, *Mt. Wilson Contr.*, No. 40, p. 7; *Astrophysical Journal*, **30,** 93, 1909.
4. From examination of the map of the sun-spot spectrum or of original plates for wave-lengths greater than 6600. Many of these lines have previously been studied at Mount Wilson, but they have been re-examined because the later plates are probably better.
5. Masked by other lines.
6. This line (according to the accurate measures of Schultz (*Zeit. für Wiss. Phot.*, **11,** 209, 1912) lies at 5535.755 on Rowland's scale, and falls in a very confused region between the iron line 5535.644 (intensity 2) and the line 5535.778 (intensity o), for which Rowland gives no origin. Between these two there is a faint line which is confined to the spot, in which it is of intensity o and shows Zeeman effect. It was measured and found to be at 5535.703. There is also a faint band line close by showing no Zeeman effect, but nothing which can be identified as the barium line. It is either absent, or masked by 5535.778.
7. Too far to the violet to show changes in the spot spectrum.
8. Rowland's line 3076.002, for which he gives no origin. Kayser and Runge find 3075.99 in the arc; Exner and Hascheck, 3076.02.
9. But little weakened in the photographic map. Weakening confirmed on an original negative. Zeeman effect stronger over penumbra of spot than over umbra. May be blended with a band line. These are numerous in this region.

(except for magnesium) which is not known to be the case for any other elements. They are also noteworthy for possessing two radiation potentials, corresponding to the shift of an electron from its normal position in the neutral atom to the position corresponding to the limits of the subordinate series of single lines and of triplets.

These lines afford the most delicate tests for the presence of the ionized or neutral atoms of these elements. Their behavior in spots is indicated in Table II. Various other lines of these elements and the observable lines of zinc are included.

This table exhibits the well-known fact that the strongest lines of the alkaline earths (except magnesium) are enhanced lines.

The series of calcium and magnesium, both of triplets and of single lines, are conspicuous in the solar spectrum, and only some of the more prominent lines are given in the table. The calcium lines are considerably strengthened in the spot spectrum; those of magnesium are not, with one exception.

In the case of strontium, only the strongest lines of the neutral atom appear in the sun; these are very faint, but are intensified in spots, while not one of the strong low-temperature lines of barium appears either in the sun or the spots. This has been confirmed by examining a number of those which did not belong to known series and are therefore not given in the table.

3. APPLICATION OF THE THEORY OF IONIZATION TO THESE DATA

The ionization potentials of a number of elements of astrophysical importance are as follows:

	Volts		Volts		Volts
Caesium	3.81	Barium	5.12	Ba+	9.86
Rubidium	4.16	Strontium	5.67	Sr+	10.70
Potassium	4.32	Calcium	6.08	Ca+	11.86
Sodium	5.11	Magnesium	7.65	Mg+	15.02
Lithium	5.37	Zinc	9.4	Zn+	18.20
Hydrogen	13.54	Helium	25.2	He+	53.4

The last column gives the potentials corresponding to the additional energy required to remove a second electron after the first is already gone.

We may now proceed to calculate the percentages of the atoms of various elements which are neutral, singly-ionized, and doubly-ionized, in a gaseous mixture at the temperature of the reversing layer and of a sun-spot—which we will take in round numbers as 6000° and 4000° respectively—and at various pressures.

Since the pressure in the solar atmosphere is unknown within very wide limits (except that it is probably small) we may save trouble by computing for assigned values of the "effective pressure" P_1, for sodium, using equations (2) and (7). The difference between the effective pressure and the true pressure will be much less than the range of uncertainty of the latter. We may then find the percentages of ionization of other elements by (5) or (11).

The results are given in Table III. Neutral atoms are denoted as usual by Ca; singly-ionized, by Ca+; and doubly-ionized, by Ca++ (taking calcium as an example). The percentage of atoms in the highest stage of ionization for each element is not given, except for hydrogen, since it can easily be found by subtracting the sum of the others from 100, and also because atoms of the elements given in the table, when in these states, absorb no known lines in the visible spectrum.

It should be remembered that the equations from which these tables have been calculated are based upon certain assumptions, as yet unverified, namely, that the chemical constant for the electron may be derived by an extrapolation of the Sackur-Tetrode relation (which gives this constant as a function of the atomic weight of an element) to the electron, considered as having an atomic weight of 0.00055, and also that the chemical constant is the same for the neutral and ionized atoms of any element.[1] An error in the former assumption could be corrected by multiplying all the tabular pressures by a constant factor; one in the second, by multiplying the pressure for each element by an appropriate factor before entering the table.

What is more important, the tabular numbers depend on the assumption that temperature alone is effective in producing ionization. This is certainly not the case in the solar atmosphere, where the absorption of radiation, though it does not completely ionize the absorbing atoms, must undoubtedly get them into a state in

[1] See Saha, *Philosophical Magazine*, **40**, 479, 1920.

which ionization by some other stimulus becomes easier.[1] This should operate to increase the ionization in the reversing layer to a considerable degree.

TABLE III

PERCENTAGES OF NEUTRAL AND IONIZED ATOMS

ELEMENT	P IN ATMOSPHERES						
	1	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}
	Reversing Layer—Temperature 6000° C.						
Cs.....	22.6	6.1	0.87	0.09	0.01	0.001	0.0001
Rb.....	37.2	12.3	1.7	0.18	0.02	0.002	0.0002
K.....	44.7	14.8	2.2	0.19	0.02	0.002	0.0002
Na.....	79.3	44.4	9.6	1.1	0.11	0.011	0.0011
Li.....	86.1	57.0	15.0	1.9	0.19	0.019	0.002
Ba.....	79.1	45.0	9.7	1.1	0.10	0.005	0.0001
Ba+....	10.9	55.0	90.2	98.0	91.3	48.3	9.6
Sr.....	91.7	70.2	23.8	3.2	0.3	0.03	0.001
Sr+.....	8.3	29.8	76.2	96.4	97.9	84.6	35.8
Ca.....	96.1	84.0	41.0	6.8	0.7	0.07	0.007
Ca+....	3.9	16.0	59.0	93.2	99.1	98.0	83.6
Mg.....	99.8	99.1	93.5	60.4	13.2	1.5	0.15
Mg+....	0.2	0.9	6.5	29.6	86.8	98.5	99.8
Zn.....	99.99	99.96	99.7	96.9	75.6	23.7	3.1
Zn+....	0.01	0.04	0.3	3.1	24.4	76.3	96.9
H+.....	2×10^{-6}	1×10^{-5}	8×10^{-5}	8×10^{-4}	0.008	0.08	0.76
	Sun-Spot—Temperature 4000° C.						
Cs.....	54.9	27.0	9.3	2.2	0.30	0.032	0.003
Rb.....	77.0	50.0	21.9	5.7	0.83	0.089	0.009
K.....	84.2	61.5	30.9	8.8	1.3	0.14	0.014
Na.....	98.1	94.1	81.5	48.9	11.6	1.37	0.14
Li.....	99.2	97.1	90.3	66.9	21.8	2.95	0.29
Ba.....	98.2	94.1	81.8	49.5	11.8	1.42	0.14
Ba+....	1.8	5.9	18.2	50.5	88.2	98.58	99.86
Sr.....	99.63	98.76	95.8	82.9	40.0	6.6	0.72
Sr+.....	0.37	1.24	4.2	17.1	60.0	93.4	99.28
Ca.....	99.89	99.62	98.7	94.1	68.5	18.7	2.3
Ca+....	0.11	0.38	1.3	5.9	31.5	71.3	97.7
Mg.....	99.999	99.996	99.975	99.93	99.52	95.6	69.0
Mg+....	0.001	0.004	0.025	0.07	0.48	4.4	31.0
Zn.....	100.00	100.00	100.00	100.00	99.995	99.95	99.51
Zn+....	1.3×10^{-5}	4.3×10^{-5}	1.6×10^{-4}	7.2×10^{-4}	0.005	0.05	0.49
H.+....	4.7×10^{-11}	1.6×10^{-10}	5.7×10^{-10}	2.6×10^{-9}	1.9×10^{-8}	1.8×10^{-7}	1.8×10^{-6}

In applying these results to the interpretation of the spot spectrum, it is probably safe to work on the assumption that the

[1] The existence of such an effect for iodine has been experimentally proved by Smythe and Compton (*Physical Review*, **16**, 501, 1920). Its importance in the solar atmosphere was suggested to the writer, in conversation, by Dr. Lindblad.

more abundant elements in the solar atmosphere are present in amounts much greater than is necessary to produce strong lines and that, for the principal lines of these elements (at least) the absorption is substantially "saturated" so that a considerable change in the amount of element would produce little change in these lines, except in the strength of the wings. It is also important to remember that, according to modern theory, the atoms of a given element, in their normal state, are ready to absorb certain fundamental lines (those of the principal series, denoted by 1s–2p, etc., above) while, in order that they may absorb other lines (such as those of the subordinate series) they must first have absorbed energy enough to shift the electrons within them to some other definite state, less stable than the normal one. Saha, who has discussed this matter in detail, states that the relative numbers of atoms which are in condition to absorb lines of these various groups probably depend upon the temperature alone—high temperature favoring the absorption of the subordinate lines. It is obvious that "high" temperature must here be a relative term, the scale for any element being roughly proportional to the temperature at which ionization becomes large.

The main features of the behavior of the lines of the alkali metals in spots, as Saha has shown, may be accounted for by the relative ease of ionization, sodium being considerably ionized, potassium still more, and rubidium completely so, except in spots. But the fact that lithium, like rubidium, appears only in the spots, cannot be explained on the basis of ionization potentials alone, for it is harder to ionize than sodium, and the percentage of neutral atoms should be greater.

A solution may be found in the probable difference in the relative abundance of the elements. Sodium and potassium are among the most abundant of terrestrial metals, being present in the earth's crust in nearly equal amount (each about 2 per cent of the whole). Lithium is relatively rare on the earth, and, if the proportion present in the solar atmosphere is sufficiently small, it may well be that the number of neutral atoms which remain is insufficient to produce perceptible absorption lines, except in the spots, where the ionization is less. Among the far more numerous

sodium atoms the number which escape ionization, though a smaller percentage of the whole, is so great that there are enough of them in a condition to absorb the subordinate series fairly strongly, and the principal series heavily. In potassium the percentage of neutral atoms is much less, and only the lines of the principal series appear, with moderate intensity, but much strengthened in the spots.

It remains somewhat puzzling, however, why rubidium, which is much easier to ionize than lithium, should show nearly the same behavior in the spot spectrum. According to Table III the percentage of neutral rubidium atoms should be less than one-tenth as great as in the case of lithium. The increase in this percentage in spots is great enough for both elements to account for the appearance of their lines there, and not on the disk, provided that we assume that the pressure in the region where the lines are absorbed is 0.01 atmospheres or less (which seems reasonable). But it is hard to see why the rubidium lines should appear at all, unless it is considerably more abundant in the sun than lithium is, which does not at all seem to be the situation on the earth.

Passing to the alkaline earths we find improved opportunities. While the ionized atoms of the alkali metals give no recognized lines in the visible spectrum, those of calcium and its congeners are unique in that the fundamental lines of the ionized atom lie within the region accessible to astrophysics. The great importance of these lines (H and K of calcium and λλ 4078, 4215 of strontium) is thus explained. They enable us to detect the ionization of these easily ionized atoms as soon as it occurs. The ionized atom of calcium is at once capable of absorbing the H and K lines.

From Table III it appears that, under the pressures which probably prevail in the reversing layer, the greater part of the calcium is in the singly-ionized condition.

H and K, therefore, are the lines which the atoms of one of the most abundant constituents of the solar atmosphere absorb most powerfully, when in that state in which under solar conditions they are most likely to occur. There are no other lines in the visible spectrum, so far as known, for which these three maximal conditions are all satisfied.

It is not surprising, therefore, that they are exceptionally conspicuous, and that few other lines play so important a part throughout almost the whole of the sequence of stellar spectra. A great deal of the calcium must be ionized, even in the spots, and this, coupled with the fact that the absorption in the H and K lines is probably far more than saturated, makes it easy to understand why these lines are not perceptibly weakened in the spots. The same remark applies, with even greater force, to the corresponding enhanced lines of strontium and barium.

Magnesium should also be considerably ionized in the reversing layer; but the only line by which we can detect the presence of ionized magnesium (λ 4481) is absorbed only by atoms which have undergone two internal changes subsequent to ionization, corresponding to the shifting of an electron from its normal position to the states denoted symbolically by 2p and 3d, and demanding the absorption of large amounts of energy. The temperature of the reversing layer, compared with that at which a considerable percentage of these Mg+ atoms would lose a second electron, is undoubtedly low. Hence very few such atoms indeed should be in a state to absorb λ 4481, and it is not surprising that it is excessively faint in the solar spectrum, if it appears at all, or that it becomes conspicuous only in the hotter stars.

If the earth's atmosphere transmitted light of short enough wave-length, we would probably find that the fundamental pair of enhanced magnesium lines near λ 2800 were rivals of H and K in intensity and astrophysical importance.

The very strong infra-red lines of calcium at λλ 8498, 8542, 8662, belong to the diffuse series and represent a stage of excitation intermediate between H and K and λ 4481. King[1] has found that they cannot be produced in the furnace, at temperatures at which H and K appear strongly. It may safely be predicted that they are weakened in the spot spectrum, and the matter will be investigated when suitable photographs can be obtained. The corresponding group for strontium is hopelessly far in the infra-red; but the leading

[1] *Mt. Wilson Contr.*, No. 150; *Astrophysical Journal*, **48**, 13, 1918.

pair of the sharp series of enhanced lines is accessible. One line is concealed by a blend; the other is greatly weakened in spots.

For barium, which is easier to ionize a second time, the enhanced lines of the diffuse series are not perceptibly weakened in spots.

The behavior of the lines due to the neutral atoms is also in accordance with this theory. Beginning with zinc, which has the highest ionization potential, we find that the prominent triplet in the blue is greatly weakened in spots, and the line in the red is also weakened, though to a less degree. These lines belong to the sharp series of triplets and the diffuse series of single lines, both of which are absorbed by atoms in which the electron has been shifted one step from its normal position (to 2p or 2P). Measured by a scale appropriate to the zinc atom, the temperature of the photosphere must be low and that of the spots very low; hence the weakening of these subordinate lines is just what should be expected.

It is of interest to note that the lines of hydrogen, which also belong to a subordinate series and to an element for which the ionization potential is high, are much weakened in the spot spectrum.[1]

The lines of magnesium are very little affected in spots. In this case the effects of higher excitation and greater ionization at the higher temperature of the reversing layer appear very nearly to balance one another except in the case of λ 4571, which is a characteristic low-temperature line, and is the easiest of all magnesium lines to excite in the laboratory.[2] This line, as might be expected, is strengthened in the spots, but only to a moderate degree.

For calcium, all the important lines of the neutral atom are strengthened in spots, indicating that the controlling factor is the diminution of ionization at the lower temperature, which increases the number of neutral atoms. Where allowance is made for the general increase toward the red of the degree to which lines are affected in spots, it appears that the strengthening is greatest for the lines absorbed by atoms in which the electron is in the normal state, 1S; less for those of the subordinate series, in which the

[1] Adams, *Mt. Wilson Contr.*, No. 40, p. 10; *Astrophysical Journal*, **30**, 95, 1909.

[2] Foote and Mohler, *Philosophical Magazine*, **37**, 40, 1919.

electron starts from the state 2P (for single lines) or 2p (for triplets); and least of all for those of the Bergmann or combination series in which it starts from 3D or 3d. According to Saha[1] the ratio of the number of atoms which are in any one of these states to the number which are in the preceding stage should increase with the temperature. Hence the effect of the diminished number of neutral atoms in the hotter regions is almost neutralized for the lines of the "outer" series (3D–4F and 3D–2P) and increased for the fundamental lines (1S–2P), and 1S–2p.[2] The first of these, λ 4227, is so far to the violet that the effect in spots is not conspicuous, though great for a line in this region. This is very prominent in the cooler stars. The second, λ 6573, is one of the most remarkable of all sun-spot lines. It is an exact analogue of λ 4571 of magnesium, which likewise shows the greatest strengthening of any line of the element to which it belongs.

For strontium, the ionization is still greater, and the amount of the element present in the solar atmosphere probably much less, if we may judge by the earth's crust, in which strontium and barium are much rarer than calcium and magnesium. It is not surprising, therefore, that only the strongest lines of the neutral atom appear in the solar spectrum, or that these should be strengthened in the spots, especially the fundamental line λ 4607. λ 6892, which is analogous to λ 6573 of calcium, is absent even in the spot spectrum.

The probable reason for this is that the temperature, even in the spots, is high on the scale which it is necessary to use for the neutral strontium atom, and that, under this high excitation, the proportion of atoms in which the electron undergoes the corresponding change is small.

For barium the final stage appears to have been reached. The lines of the neutral atom do not appear at all either in the spectrum of the sun or of the spots. Even the fundamental line 1S–2P (λ 5535) is gone. This can be explained only on the assumption that barium is completely ionized in the sun, leaving not a trace of the neutral atoms, for the element is abundant enough, as is shown by the strength of its enhanced lines.

[1] *Proceedings of the Royal Society*, A, **99**, 142, 1921.

From the qualitative standpoint this explanation is quite satisfactory, but a quantitative study brings out difficulties. The ionization potentials for sodium and barium, as determined from their spectral series, should be practically identical. Hence, by the theory developed above, the atoms of both should be ionized to the same degree. But the direct spectroscopic evidence indicates that barium is ionized to a much greater extent than sodium. The absence of the flame lines even in spots, and the fact that the enhanced lines of the diffuse series are not sensibly weakened, would indicate a decidedly lower ionization potential both for Ba and Ba+.

A similar discrepancy, though less marked, exists in the case of strontium.

As the theory of the subject is still in an early stage of development, these facts may be taken as the results of an experiment performed by nature upon a grand scale, which may be used in improving our theories; but speculation as to the exact form of this improvement would at present be premature.

4. GENERAL COMMENTS

The data detailed above cover all the cases in which elements, for which the ionization potentials and spectral series are known, possess lines of any notable strength within the region in which the sun-spot spectrum can be satisfactorily investigated. It is evident that they are all in striking agreement with Dr. Saha's theory of ionization and that they confirm it in a very conclusive fashion, both in the matters in regard to which he has made predictions, and in various others which are here discussed for the first time.

There are many other elements whose lines show conspicuous changes in the spots; but no data are at present available regarding the ionization potentials. Most of these elements have spectra rich in lines, for which the series relations are known very incompletely or not at all.

The behavior of these lines in the spot spectrum,[1] coupled with that in the furnace, arc, and spark, makes it possible to arrange them in a decidedly definite order.

[1] Adams, *Mt. Wilson Contr.*, No. 40, pp. 6–26; *Astrophysical Journal*, **30**, 91–111, 1909.

A few of the elements already discussed are added for comparison. Lines which are relatively strong at the lowest temperatures (King's Class I) are called flame lines for brevity.

Calcium.—All lines strong in sun. Flame lines much strengthened in spots. Arc lines considerably strengthened in spots. Enhanced lines produced at medium temperature in furnace,[2] very strong in arc, and not weakened in spots.

Scandium.[1]—Flame lines very faint or absent in sun, greatly strengthened in spots. Arc lines faint in sun, much strengthened in spots. Enhanced lines produced in furnace at high temperature, strong in arc, and slightly weakened in spots.

Titanium[2] *and Vanadium*[3]—These elements behave in a very similar fashion, both in the laboratory and in sun-spots. The furnace lines are greatly strengthened in spots and the arc lines strengthened. Practically all the arc lines may be obtained in the furnace. The enhanced lines can be obtained in the furnace only with great difficulty, and are weakened in spots.

Iron,[4] *manganese,*[5] *and chromium*[3].—These again are similar in their behavior. The flame lines are strengthened in the spot spectrum, the arc lines little affected, and the enhanced lines much weakened. Many arc lines are very hard to produce in the furnace, and the enhanced lines cannot be produced there at all, and are faint in the arc. The percentage of lines which are strengthened in the spot spectrum is greater for chromium than for manganese and still smaller in the case of iron.

Cobalt and nickel.[6]—The number of lines weakened in spots is nearly as great as that of the strengthened lines in the case of cobalt, and much greater for nickel. The number of arc lines which cannot be obtained in the furnace is greater for nickel than for cobalt.

[1] King, *Mt. Wilson Contr.*, No. 214; *Astrophysical Journal*, **54**, 28, 1921.

[2] King, *Mt. Wilson Contr.*, No. 76; *Astrophysical Journal*, **39**, 139, 1914.

[3] King, *Mt. Wilson Contr.*, No. 94; *Astrophysical Journal*, **41**, 86, 1915.

[4] King, *Mt. Wilson Contr.*, No. 66; *Astrophysical Journal*, **37**, 239, 1913.

[5] King, *Mt. Wilson Contr.*, No. 198; *Astrophysical Journal*, **53**, 133, 1921.

[6] King, *Mt. Wilson Contr.*, No. 108; *Astrophysical Journal*, **42**, 344, 1915.

A number of cobalt lines and one nickel line described by Lockyer as enhanced appear in the furnace, but King gives good reason for supposing that these are not enhanced lines of the regular type.

Copper reveals itself in the sun by the first pair of the principal series, λλ 3274, 3247, which are produced when mere traces of the metal are present. They are recorded as of intensity 10 by Rowland, yet no trace of the lines of the subordinate series appears; although λλ 5782, 5700 are furnace lines.[1] The limit of the principal series, 1605 A,[2] corresponds to an ionization potential of 7.7 volts.

For *zinc*, as has already been noted, the arc lines are fairly strong in the sun, but greatly weakened in the spots. Finally, at the end of the series, come *silicon*, for which all the solar lines are greatly weakened in spots, and *oxygen*. The triplet of the latter in the extreme red, λλ 7772–7775, is faintly present in the arc, stronger in the spark, and, though fairly strong in the sun, is completely obliterated in the spot spectrum.[3] Yet the series relations of these lines, which belong to a combination series 2s–mp, show conclusively that they are produced by the neutral atom.

Part of the effect observed in the spots, however, is probably due to the formation of titanium oxide, which must diminish considerably the amount of free oxygen in the solar atmosphere.

The foregoing list shows a definite sequence in the behavior of the elements, alike in furnace, arc, and sun-spots, which suggests strongly that we have to do with a series in which the ionization potential steadily increases (except perhaps in the case of copper, for which the data are very meager). It cannot be a matter of chance that the list, from calcium, and, indeed, from potassium to zinc in order of increasing difficulty of spectroscopic excitation, is a complete list of the intervening elements in the order of their atomic numbers. It appears highly probable that in this region at least the ionization potential increases with the interval in the periodic table which separates the element from the preceding

[1] King, *Astrophysical Journal*, **21**, 250, 1904.

[2] Konen, *Das Leuchten der Gase* (Braunschweig, 1913), p. 146.

[3] Merrill, *Mt. Wilson Contr.*, No. 183, p. 7; *Astrophysical Journal*, **51**, 247, 1920.

noble gas. Whether this relation holds good more generally cannot be known until further data become available, but it is of interest to note that silicon and oxygen, which are evidently decidedly hard to ionize, fit in very well, since, in the known cases, the ionization potential falls steadily for corresponding elements in each successive period of the periodic table.

The behavior in the spot spectrum of most of the elements in this list is intermediate between that of calcium and that of zinc. It may therefore be surmised that the ionization potentials for these metals lie between six and nine volts, as Saha has suggested[1] without specifying his reasons.

One further comment may be made on the foregoing. It is evident that the enhanced lines of an easily ionized element, such as barium or calcium, may be produced with a weaker excitation than the arc lines of an element of difficult ionization, such as zinc or silicon; the intervening elements present all degrees intermediate between these extremes. The mere fact that an enhanced line appears in a given spectrum is therefore not a proof that the temperature, for example, is high on an absolute scale, but only that it is high relative to the scale appropriate for the element in question. It would probably be fairly safe to assume that temperatures proportional to the various ionization potentials produce similar effects upon different atoms, but further knowledge is necessary before we can be sure of this.

The principles of the ionization theory will evidently be of great importance throughout the whole field of astrophysics, and Dr. Saha[2] has made an application of the highest interest to the question of the physical meaning of the sequence of stellar spectra. The writer hopes to present a discussion of many features of this problem at some time in the not distant future.

The possibilities of the new method appear to be very great. To utilize it fully, years of work will be required to study the behavior of the elements mentioned above and of others, in the stars, in laboratory spectra, and by the direct measurement of

[1] *Philosophical Magazine*, **40**, 822, 1920.

[2] *Proceedings of the Royal Society*, A, **99**, 135, 1921.

ionization, but the prospect of increase of our knowledge, both of atoms and of stars, as a result of such researches, makes it urgently desirable that they should be carried out.

MOUNT WILSON OBSERVATORY
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NOTE.—An admirable summary of the theory of ionization by Mr. E. A. Milne, in the *Observatory* for September (**44**, 261, 1921), has appeared since the completion of the foregoing discussion. He points out the influence of the free electrons resulting from the dissociation of easily ionized atoms in diminishing the ionization of other atoms, and gives the correct equation for second-stage ionization. Section 1 of the present paper was prepared independently and carries the analysis somewhat further, so that it appears sufficient to call attention here to Mr. Milne's prior publication.